Cancellation of infrared divergences in trace of gluon polarization tensor at $T\neq 0$ in covariant gauges

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Perturbative infrared divergences cancel out algebraically in the trace of the transverse components of the two-loop gluon polarization tensor in covariant gauges at $T\neq 0$. The limit $\Pi_{ii}(k_4=0, |\mathbf{k}|\to 0)$ of this tensor is zero, regardless of the choice of gauge parameter.

The infrared limit of the trace of the transverse components of the polarization tensor of gluons (its magnetic mass) is an important qualitative characteristic of quantum chromodynamics, since it determines (by analogy with the Debye length) the screening of gluomagnetic forces and strongly influences the phase diagram of the evolution of quark-gluon matter. However, the single-loop approximation[1,2] yields an identically zero value for this limit. Furthermore, the corresponding single-loop polarization tensor in the infrared region of momenta leads to a fictitious singularity of the gluon propagator.[1,3] The situation in the two-loop approximation is not yet clear, since the explicit infrared divergences of each of the two-loop diagrams of the polarization tensor have seriously complicated all the calculations and have rendered them extremely ambiguous. Nevertheless, simplifications have been found possible. Our purpose in the present letter is to show that in a calculation of the trace of the transverse components of the two-loop polarization tensor all the infrared divergences, when summed, cancel out. This fact holds in all covariant α gauges.

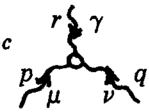
The quantum action of an $SU(N)$ gauge theory (and, in particular, of gluodynamics) has the standard form[4]

$$S=-\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}-\frac{1}{2\alpha}(\partial_\mu V^a_\mu)^2+\bar{C}_a\nabla^{ab}_\mu\partial_\mu C^b, \tag{1}$$

where $\nabla^{ab}_\mu=\delta^{ab}\partial_\mu+gf^{acb}V^c_\mu$ and $\alpha=1$ corresponds to the Feynman gauge. We have chosen a Euclidean metric in (1), and the bare propagators of the gauge field and the fictitious particles are defined in the usual way.[2] We will write out the relative signs and the functional form of the bare vertex functions explicitly (because of their importance):

$$\Gamma^{(0)}_H(p,q|r)^{abc}_\mu=-igf^{abc}$$


$$=-igf^{abc}q_\mu, \tag{2}$$

$$\Gamma^{(0)}_3(p,q,r)^{abc}_{\mu\nu\gamma}=-igf^{abc}$$


$$=-igf^{abc}[\delta_{\mu\gamma}(r-p)_\nu+\delta_{\mu\nu}(p-q)_\gamma+\delta_{\gamma\nu}(q-r)_\mu],$$

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[since several diagrams which arise in calculations of the polarization tensor in higher loops depend on the relative signs of the quantities which are specified in (2)].

The exact polarization tensor can be represented as a set of five diagrams,[5]

(3)

where all the lines correspond to exact propagators (wavy lines to gluon propagators and dashed lines to ghost propagators), and all the blackened points correspond to exact vertices. All the functions depend separately on p_4 and $|\mathbf{p}|$, and the infrared limit is understood everywhere as $p_4 = 0$ and then $|\mathbf{p}| \to 0$.

This limit of the single-loop polarization tensor was found independently in two papers:[1,3]

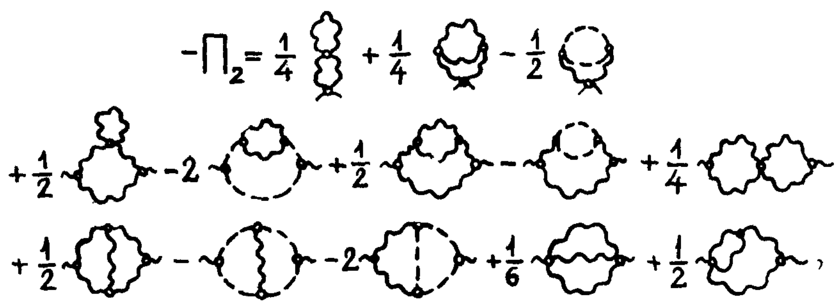
$$\Pi^{(1)}_{ij}(k_4 = 0,\ |\mathbf{k}| \to 0) = -\left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right) \frac{g^2 N |\mathbf{k}| T}{64} (9 + 2\alpha + \alpha^2), \tag{4}$$

The anomalous behavior of this limit leads to fictitious pole in the gluon propagator in the momentum region. The characteristic infrared behavior of the single-loop perturbative gluon propagator is qualitatively the same for all α gauges (see also Ref. 6 regarding the gauge $A_4 = 0$),

$$D_{ij}(p_4 = 0,\ |\mathbf{p}| \to 0) = \left(\delta_{ij} - \frac{p_i p_j}{\mathbf{p}^2}\right) \Big/ \left[\mathbf{p}^2 - (9 + 2\alpha + \alpha^2) \frac{g^2 N T}{64} |\mathbf{p}|\right], \tag{5}$$

although numerically the position of the pole depends on the choice of the values of α. This position also varies (but the pole does not disappear) when radiation corrections are made.[1,2]

We write the two-loop perturbative polarization tensor as a set of 13 topologically different diagrams,



(6)

which arise as a result of an iteration of series (3). The first seven diagrams here correspond to the iteration of lines, the next three correspond to the iteration of vertices, and the rest combine the various topological structures which arise as corrections to the vertex functions and as the first iteration of the exact two-loop diagrams of series (3).

We have calculated the infrared limit of the trace of the three-dimensional part of the polarization tensor $\Pi_{\mu\nu}$ determined by series (6). In these calculations we used the analytic-calculation program REDUCE 3.1 in the leading infrared approximation, in which each of the two sums over the fourth component of the momentum is replaced by the one term $k_4, p_4 = 0$. The three-dimensional integrals that follow are understood as repetitions and are evaluated in a dimensional regularization in the standard way:

$$\int \frac{d^D q}{(p-q)^{2\alpha} q^{2\beta}} = \pi^{D/2} \frac{\Gamma(\frac{D}{2}-\alpha)\,\Gamma(\frac{D}{2}-\beta)\,\Gamma(\alpha+\beta-\frac{D}{2})}{\Gamma(\alpha)\,\Gamma(\beta)\,\Gamma(D-\alpha-\beta)} p^{-2(\alpha+\beta-D/2)}. \qquad (7)$$

We write the result of the calculations as the algebraic sum of the individual quantities corresponding to each of the final diagrams (except for the null diagrams, 1, 5, and 8) of series (6):

$$-\Pi_{ii}^{(2)}(0) = \frac{g^2N^2}{\beta^2}\int \frac{d^3q}{(2\pi)^3}\frac{1}{|\mathbf{q}|^3}\{-\frac{1}{4}(\frac{\gamma}{2}+\frac{11}{2}) - \frac{1}{2}(\frac{\gamma^2}{8}+\frac{\gamma}{4}+\frac{1}{4}) - 2\frac{1}{16}$$

$$+\frac{1}{2}(\frac{\gamma^3}{16}+\frac{7\gamma^2}{16}+\frac{27\gamma}{16}+\frac{85}{16}) + (\frac{\gamma^2}{16}+\frac{5\gamma}{32}+\frac{7}{32}) + \frac{1}{2}(\frac{\gamma^3}{32}+\frac{\gamma^2}{2}+\frac{75\gamma}{32}+\frac{53}{16})$$

$$-(\frac{\gamma}{64}+\frac{1}{32}) - 2(\frac{\gamma}{64}+\frac{1}{32}) + \frac{1}{6}(\frac{9\gamma^3}{32}+\frac{9\gamma^2}{4}+\frac{63\gamma}{8}+\frac{81}{8}) -$$

$$-(\frac{3\gamma^3}{32}+\frac{27\gamma^2}{32}+\frac{51\gamma}{16}+\frac{9}{2})\} = 0, \qquad \gamma = \alpha - 1. \qquad (8)$$

They cancel out completely:

$$\lim_{|\mathbf{k}|\to 0} \Pi_{ii}^{(2)}(k_4 = 0, |\mathbf{k}| \to 0) = 0. \qquad (9)$$

Expression (9) shows that at $T \neq 0$ the infrared divergences algebraically cancel out in the trace of the three-dimensional part of the two-loop polarization tensor for any choice of the gauge parameter α. If the exact polarization tensor is transverse in covariant gauges (this possibility is extremely probable,[7] but it has not been proved) and is determined by simple two scalar functions,

$$\Pi_{ij}(\mathbf{k}, k_4) = (\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}) A(\mathbf{k}, k_4) + \frac{k_i k_j}{\mathbf{k}^2}\frac{k_4^2}{\mathbf{k}^2}\Pi_{44}(\mathbf{k}, k_4), \qquad (10)$$

then assertion (9) means that the limit of the function $A(k_4 = 0, |\mathbf{k}| \to 0)$ is also a null limit. Consequently, the perturbation-theory series for $\Pi_{\mu\nu}(\mathbf{k}, k_4)$ is totally freed of infrared divergences in relativistic gauges. It is thus possible to carry out reliable perturbative calculations of various quantities in the infrared momentum region. In particular, it would be very interesting to find the following terms in expansion (4) and possibly to sum this expansion, in order to reliably establish the true infrared behavior of the gluon polarization tensor without appealing to perturbation theory.

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