New confinement–deconfinement order parameter in lattice theories

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A gauge field may be in a phase of confinement or deconfinement. An order parameter, defined as the dimensionality d of the current lines of Abelian magnetic monopoles, is proposed. This definition is dictated by the confinement scenario which has magnetic monopoles condensing into a superconducting phase in which the order parameter is $d > 1$. In the deconfinement phase there is no condensation, and the order parameter is $d = 1$. This result is supported by numerical calculations.

The string tension[1] σ and the value of the Polyakov-Wilson line,[2] L, are widely used order parameters determining the phase of the gauge field (confinement or deconfinement). The string tension is the force acting between a quark and an infinitely remote antiquark. In the deconfinement phase we would thus have $\sigma = 0$, and in the confinement phase $\sigma \neq 0$. The value of L is related to the free energy F of an infinitely heavy quark in vacuum: $L = \exp\{-F/T\}$. Consequently, if such a state is realized physically ($F < \infty$), we would have $L \neq 0$. In the confinement phase we would have $F = \infty$ and $L = 0$. The order parameter which we are proposing is related to the confinement mechanism based on a model of the vacuum in which there exists a superconducting condensate of magnetic monopoles.[3] This mechanism is responsible for confinement in compact four-dimensional electrodynamics.[4] In addition, there are numerical arguments in favor of this scenario in non-Abelian lattice gauge theories.[5]

We begin by presenting the results of numerical calculations in compact four-dimensional lattice electrodynamics. In this theory, the gauge fields are determined

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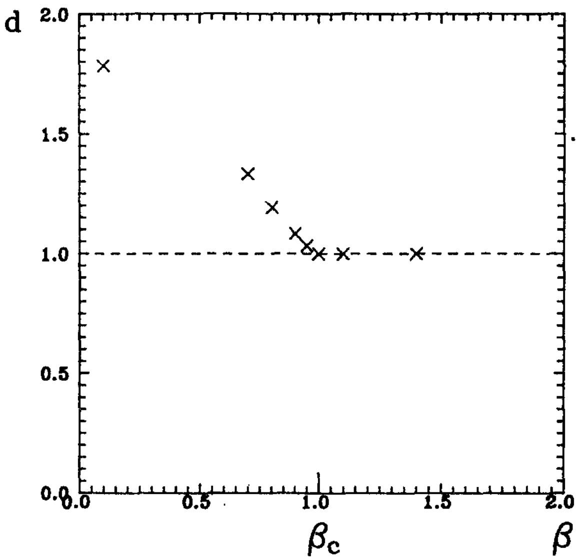

FIG. 1.

modulo 2π, so monopoles exist in a given configuration of lattice fields.[4] In 4D space, monopoles correspond to current lines which belong to the links of a dual 4D lattice (a static monopole corresponds to a current which is directed along the "time axis"). The current lines are closed by virtue of a conservation law. It has been found in numerical simulations[4,5] that in the confinement phase the current lines cover the dual lattice thickly, and many self-intersections are observed. In the deconfinement phase the current lines are far apart. It turns out that the density of lines on the lattice can be described in terms of the fractal dimensionality d. We will discuss the definition of this dimensionality below. We have generated lattice fields and distinguished the lines of magnetic currents for various values of β in a $U(1)$ lattice gauge theory. In each configuration of fields, one observes several connected entities (clusters). Figure 1 shows the dimensionality d, averaged over several configurations of fields, of the largest of the clusters for various values of β. We see that in the confinement phase ($\beta<\beta_c$) the dimensionality d is nontrivial, and at $\beta>\beta_c$ we have $d=1$. This new order parameter thus corresponds qualitatively to the picture of a condensation of magnetic monopoles in the confinement phase, with magnetic currents so close together that they have a dimensionality greater than 1.

According to our preliminary data, the selection of Abelian monopoles in an $SU(2)$ lattice gauge theory[5] leads to a similar picture. In the deconfinement phase, above a critical temperature ($T>T_c$), the dimensionality of the clusters formed by the magnetic currents is 1, while we have $d>1$ at $T<T_c$. In other words, in this case again the quantity d is an order parameter which determines the phase of the gauge field.

We define the fractal dimensionality of a line constructed from links of a lattice as the ratio of the number of links of occupied lines to the number of lattice sites through which the line runs. Since the number of links on a periodic d-dimensional lattice is larger by a factor of d than the number of sites, our definition leads to the correct dimensionality in the case in which a line occupies all links of the lattice. For lines of lower "density," this definition leads to a natural interpolation between integer dimensionalities.

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