Gravitational dipole

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Gravitational fields generated by global monopoles are discussed. In the case of a single isovector scalar field, a global monopole at rest creates essentially no gravitational field. If there are instead several scalar fields, there can be configurations which lead to gravitational fields with nontrivial properties, in particular, gravitational dipoles. These fields may be important at cosmological scales.

Upon a spontaneous breaking of global symmetry, containing the $O(3)$ group, topologically stable configurations of a scalar field or so-called global monopoles may form (see, for example, the review by Vilenkin[1]). The characteristic features of a theory which leads to such effects are described by the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\varphi^a_{,\mu})^2 - U(\varphi^a), \tag{1}$$

where φ^a is a triplet of scalar fields ($a = 1,2,3$), $\varphi^a{}_{,\mu} = \partial\varphi^a/\partial x^\mu$, and $U(\varphi^a)$ is the self-effect potential of field φ^a, which has a minimum at the point $|\varphi^a|^2 = \eta^2 \neq 0$. In simple renormalization models we have

$$U(\varphi^a) = \frac{\lambda}{4}[(\varphi^a)^2 - \eta^2]^2 . \tag{2}$$

The monopole is described by the following solution of the equations of motion:

$$\varphi^a = \eta f(r) n^a , \tag{3}$$

where $\mathbf{n}^a = r^a/r$ is a unit radius vector, and $f(r)$ satisfies the equation

$$f'' + \frac{2}{x}f' - \frac{2}{x^2}f - f(f^2 - 1) = 0, \tag{4}$$

where $x = \sqrt{\lambda}\,\eta r$, and

$$f = 1 - x^{-2} - \frac{3}{2}x^{-4} + \ldots + O(e^{-\sqrt{2}x}/x)$$

as $x \to \infty$.

The energy-momentum tensor corresponding to Lagrangian (1) is

$$T_{\mu\nu} = \varphi^a_{,\mu}\varphi^a_{,\nu} - \frac{1}{2}g_{\mu\nu}(\varphi^a_{,\alpha}\varphi^a_{,\alpha} - U). \tag{5}$$

For the monopole in (3), the energy density T_{00} at large distance is dominated by the

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differentiation of n^a:

$$T_{00} \approx \frac{\eta^2}{r^2} \quad (r \to \infty). \tag{6}$$

This behavior of T_{00} suggests that global monopoles might simulate the dark matter of the universe, since astronomical data indicate that the energy density of this dark matter behaves as in (3), with a coefficient close to the grand unification scale, $\eta \approx 10^{15}$ GeV. Unfortunately, however, it is not possible to explain the dark-mass phenomenon in terms of the energy of global monopoles, since the gravitational field of a global monopole would be vanishingly weak, despite the infinite mass. The reason is that in the weak-field limit the equation describing the gravitational potential is

$$\Delta\phi = 8\pi G (T_{00} - \frac{1}{2} T) \equiv 8\pi G \tau , \tag{7}$$

where $T = T^{\mu}_{\mu}$ is the trace of the energy-momentum tensor. According to (5), τ is given by

$$\tau \equiv T_{00} - \frac{1}{2} T = \varphi^a_{,t} \varphi^a_{,t} - \frac{U}{2} . \tag{8}$$

We see that in the steady state, with $\varphi_{,t} = 0$, the gravitation of a global monopole is determined by the potential $U(\varphi) \approx (\lambda r^4)^{-1}$ and would be of minor importance at astronomical scales. That the gravitational field of global monopoles is weak was pointed out by Barriola and Vilenkin,[2] who derived an exact solution of Einstein's equations with the source in (5), (3). It was shown that the space is essentially flat, but with a deficiency of solid angle.

In the steady state, the source described by (8) would be negative, corresponding to an antigravitation. This result does not depend on the form of solution $\varphi^a(r)$. We know, however, that a scalar field with Lagrangian (1) cannot have stationary states with a finite total energy,[3] so it is not a simple matter to construct an antigravitating state of a scalar field. Exceptions to the assertion of Ref. 3 are skyrmions,[4] which are described by a Lagrangian of fourth degree in $\varphi_{,\alpha}$. Consequently, expression (3) does not apply to them.

We note that the assertion of a gravitational attraction in the case of a positive definite energy density is valid only if this energy density falls off sufficiently rapidly at large distances and follows from an integration by parts of the expression

$$\int d^3 r r^k \partial_\alpha T^{\alpha}_{\mu} = 0.$$

Obviously, T_{00} in (6) would not allow this procedure.

We thus see that, despite the substantial energy, a global monopole is essentially incapable of exerting a gravitational effect on the surrounding matter. However, by slightly altering the model one could construct an example in which the gravitation of a global monopole is proportional to all the energy that it has. This case is realized by virtue of the familiar nonminimal coupling with the 4-curvature scalar:

$$\Delta\mathcal{L} = \zeta R(\varphi_1^a + \varphi_2^a)^2 , \quad (9)$$

where φ_1^a and φ_2^a are two isovector fields with a self-effect potential

$$U(\varphi_1, \varphi_2) = \frac{\lambda_1}{4}[(\varphi_1^a)^2 - \eta_1^2]^2 + \frac{\lambda_2}{4}[(\varphi_2^a)^2 - \eta_2^2]^2 . \quad (10)$$

An interaction of the type in (10) might arise, for example, from radiation corrections to the Lagrangian $\lambda(\varphi_1^a\varphi_2^a)(\varphi_1^b)^2$. If it does, the latter would have to be taken into account in the discussion below. However, we will not take that approach.

In a stage in which φ_2^a takes on a constant value $\varphi_2^a = \eta_2 n_0^a$, where n_0^a is a constant unit vector, and φ_1^a forms the "hedgehog" in (3), the correction to the energy-momentum transfer for interaction (9) is

$$\Delta T_{\mu\nu} = 2\zeta\eta_1\eta_2 (\partial_\mu\partial_\nu - g_{\mu\nu}\partial^2)\frac{n_0^a r^a}{r} . \quad (11)$$

As a result, there is a gravitational-field source $\Delta\tau = 2\zeta\eta_1\eta_2(n^a n_0^a)r^{-2}$ in Eq. (7), from which we find

$$\phi = 8\pi G\zeta\eta_1\eta_2 (n^a n_0^a) , \quad (12)$$

where $n^a = r^a/r$. It seems extremely curious that the axis of the dipole is fixed by the broken symmetry in isotopic space. As a result, we have derived a gravitational dipole which is repulsive in one hemisphere and attractive in the other. The corresponding force has only a θ component and is given by

$$F = -8\pi G\zeta\eta_1\eta_2 \frac{\sin\theta}{r} . \quad (13)$$

This force falls off only as r^{-1}. With $\eta = 10^{15}$ GeV, the gravitational force of such a monopole at a distance of 100 kpc would be on the order of the gravitational force of 10^{11} solar masses. Such an entity would focus particles incident on itself, forming a filamentary wake jet. This effect might play a role in shaping the large-scale structure of the universe.

Since the force of the (nongravitational) interaction of two global monopoles does not fall off with the distance,

$$F = -\frac{\partial M}{\partial R} \sim \eta^2 ,$$

where $M(R)$ is the mass of the monopole-antimonopole pair at a distance R, such pairs should collapse in a time on the order of R. (The equation of motion is $\ddot{R} = -R^{-1}$.) We would thus expect that at each instant there would be one monpole in the universe, with a Hubble size $l = H^{-1} \sim t$. Under the assumption that the structure formed after the recombination of hydrogen, i.e., with a redshift $z \approx 10^3$, we find something on the order of 10^9 structure condensation centers at the scale of the contemporary horizon. This result seems reasonable.

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