Analytic structure of the amplitude of *dt* scattering near the elastic threshold

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A model-independent description of *dt* and $n\alpha$ scattering has been obtained by means of an effective-radius expansion. The fusion reaction $dt \to n\alpha$ near the elastic threshold has also been obtained using an effective-radius expansion. The positions of the *S*-matrix poles, which are associated with the existence of an $^5He^*(3/2^+)$ resonance and with the Coulomb interaction, have been found.

1. The reaction $^3H(dn)^4He$ with a large energy release, ≈ 17.6 MeV, is important in nuclear fusion and muon-catalysis reactions. In Refs. 1 and 2 the cross sections of this reaction were measured within $\sim 1.5\%$ near the resonance $^5He^*(3/2^+)$ and were analyzed by the *R*-matrix method. The results of that analysis were used to determine the positions of the resonance pole *M* and the pole *S*, which was interpreted as the "shadow" pole associated with the $n\alpha$ channel.[2,3]

We will use, after some adjustments, the model-independent method, which was used previously in a Coulomb short-range-interaction problem,[4–8] to describe the *dt* scattering and to study the analytic structure of the *S* matrix near the elastic threshold. We obtained the complex scattering lengths a_{cs} and the effective radius r_{cs} which describe all the experimental data at low energies. In addition to the pair of poles with the energies $E_R = (47 - i\,36)$ keV and $E_{R'} = (77 + i14)$ keV, we found two series of Coulomb poles which converge at the point $E = 0$.

2. The effective-radius approximation for *dt* scattering. The *S* matrix has the form $(l = 0)$:

$$S_0(k) = e^{2i\sigma_0} \frac{\cot\delta_{cs} + i}{\cot\delta_{cs} - i} = e^{2i\sigma_0} \frac{a(k) - ib_-(k)}{a(k) - ib_+(k)}. \tag{1}$$

Here we use the effective-radius approximation for the nuclear Coulomb scattering phase $\delta_{cs}(k)$[1)]

$$D\cot\delta_{cs} + 2\zeta h(\zeta/k) = K_{cs}(k^2) = -\frac{1}{a_{cs}} + \frac{1}{2} r_{cs} k^2, \tag{2}$$

where $D(k)$ is the Coulomb barrier penetration; the remaining quantities are described in Ref. 4, for example; $K_{cs} \equiv \alpha(k^2) - i\beta(k^2) \approx \alpha_0 + \alpha_1 k^2 - i(\beta_0 + \beta_1 k^2)$ and $a(k) = \alpha(k^2) - 2\zeta h$, $b_\pm(k) = \beta(k^2) \pm D$. We note that $K_{cs}(k^2)$ is an analytic function of k^2 which has singularities at the inelastic channel thresholds (the unitarity reveals that $\beta \geqslant 0$).

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Since the fusion reaction proceeds from the s wave in the entrance dt channel through the resonance $^5He^*(3/2^+)$ (the contributions from other states amount to $<1\%$; see Ref. 9), the cross section for the reaction can be written in terms of the parameters of the elastic dt scattering:

$$\sigma_r(E) = \frac{2\pi}{3k^2}(1-|S_{dt}|^2) = \frac{4\pi D(k)s(E)}{3E}, \quad s(E) = \frac{\beta(k^2)}{a^2(k)+b_+^2(k)}, \tag{3}$$

where $s(E)$ is the so-called astrophysical function, and $E=k^2/2$. Using the data of Refs. 1 and 2 on the reaction cross section (25 experimental data points), we obtained the best set of low-energy parameters of the dt system

$$\alpha_0 = 0{,}233, \quad \beta_0 = 0{,}0785, \quad \alpha_1 = 0.121, \quad \beta_1 = 7{,}98\cdot 10^{-3}. \tag{4}$$

($\chi^2 \approx 0{,}3$, see Fig. 1; compare with that in Ref. 8).

In this case we have $s(0)=1.295$, and a conversion to the scattering length and effective radius yields

$$a_{cs} = -(92{,}4+i31{,}1)\ \text{fm}, \qquad r_{cs} = (5{,}79-i\ 0{,}383)\ \text{fm}. \tag{4'}$$

These parameters describe, within the experimental error, both the elastic scattering and the fusion reaction at $E\leqslant 150$ keV.

3. Analytic structure of the S matrix. The position of the real, virtual, and quasi-

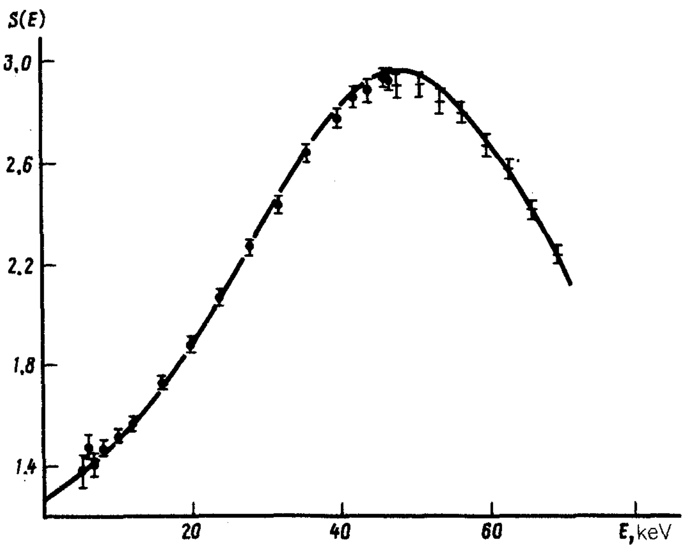

FIG. 1. The astrophysical function $S(E)$. The solid curve was constructed on the basis of Eq. (3) using the parameters in (4); †—Experimental data taken from Ref. 2; ●—experimental data of Ref. 1.

stationary levels is determined by the equation[4-7]

$$f_0 \equiv \lambda - 2\zeta [\psi(1+\zeta/\lambda) + \ln(\lambda/|\zeta|)] = \frac{1}{a_{cs}} + \frac{1}{2} r_{cs} \lambda^2 , \qquad (5)$$

where $\lambda = -ik$, and $f_0(\lambda)$ is a real-valued analytic function λ.

In the absence of absorption the S-matrix poles are arranged symmetrically with respect to the imaginary axis in the k plane. The nuclear interaction disrupts this symmetry and removes the Coulomb poles, $k_n = i\zeta/n(\zeta > 0,\ n = 1,2,...)$, from the imaginary negative semiaxis. At resonance, with $\mathrm{Re}\, a_{cs} < 0$, there are two poles close to zero, with $\mathrm{Im} k < 0$. A numerical solution of Eq. (5) with the parameters (4) gives

$$k_R = 1{,}33 - i\ 0{,}45, \qquad k_{R'} = -1{,}61 - i\ 0{,}15 \qquad (6)$$

Equation (5), moreover, evaluates the Coulomb series[2)] of poles ($k_n = -i/\nu_n$):

$$\nu_n = n + \nu_0 + c/(n+\nu_0)^2 + O(n^{-4}), \qquad -1/2 < \mathrm{Re}\,\nu_0 < 1/2,$$
$$\nu_0 = \frac{1}{2\pi i}\ln(1-2iA), \qquad c = (1 - 3r_{cs}/a_B)A^2/12\pi^2(1-2iA), \qquad (7)$$

where $A = 2\pi a_{cs}/a_B$.

The poles of the dt scattering amplitude, which are situated near the elastic threshold, are shown in Fig. 2. In the absence of absorption the potentials with a barrier always have a pair of symmetric poles.[10] The solid curves in Fig. 2 show the paths of these poles when α_0 is varied (here we assumed $\beta_0 = r_{cs} = 0$). We marked

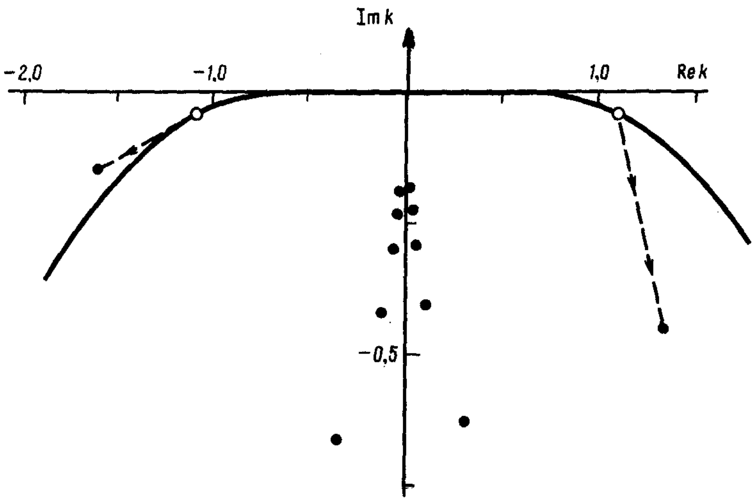

FIG. 2. The poles of the dt scattering amplitude in the complex k plane. O—Position of the poles ignoring absorption; ●—position of the poles taking the absorption into account.

the positions of the poles $\alpha_0 = 0.233$ [cf. with (4)]. The introduction of absorption, i.e., allowance for the $n\alpha$ channel, shifts the poles considerably, although no changes occur qualitatively. Upon further increase of the absorption the poles R' may enter the upper half-plane, $\mathrm{Im}k > 0$.

For the resonant part of the elastic $n\alpha$ scattering [cf. with (1)] we have

$$S_{n\alpha} = e^{2i\varphi}[a(k) + ib_-(k)]/[a(k) - ib_+(k)] \tag{8}$$

(ϕ is the potential phase). Although the positions of the S-matrix poles are the same in each channel, the energy dependences of the cross sections differ and $\sigma_{n\alpha}(E)$ depends appreciably on the phase. The statement made in Ref. 3 that the positions of the maxima in the cross sections for the $n\alpha$ collision and for the reaction $dt \to n\alpha$ are determined directly by the positions of the poles M and S is therefore unjustifiable. This topic will be discussed in greater detail in a separate paper.

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[1)] Below $e = \hbar = m = 1$ and $\zeta = z_1 z_2$; the energy unit for the dt system is $E_C = 59.9$ keV and the Bohr radius is $a_B = 24.0$ fm, and $\zeta = 1$.

[2)] A series of Coulomb poles in the left half-plane k is described by similar expressions.

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