Electron–ion collisions in laser fields at relativistic intensity

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Electron–ion collisions in a laser wave of relativistic intensity are analyzed in the approximation of a given classical trajectory for the electron. The equivalent-photon method is used to derive the bremsstrahlung cross section and also the cross section for the excitation of the ion.

The intensity of laser sources has now reached $I_1 = 10^{17}$ W/cm^2 (Ref. 1). In the future, we will apparently see values higher than $I_2 = 10^{19}$ W/cm^2 (Ref. 2). At such field levels at frequencies in the optical range, there can be effective excitation of multiply charged electrons by electrons, electrostimulated nuclear fission,[3] and also the production of electron-positron pairs in plasmas in collisions of electrons with nuclei.[4] Theoretical work shows that these processes can occur at probabilities high enough to allow detection.

A complete quantum-mechanical calculation on these processes has yet to be carried out. The existing estimates are based on qualitative arguments that the basic characteristics of high-energy processes involving ultrarelativistic electrons are determined by the total energy of the electron in the wave field.

The purpose of the present study is to present the case for the use of a method based on the approximation of a given classical trajectory. We will also calculate the cross sections for resonant bremsstrahlung and target excitation in the scattering of ultrarelativistic electrons in ultrastrong fields of intensity $I > I_2$.

Let us examine processes in which the total energy of the electron in the field of the electromagnetic wave changes by a small amount ΔE, so we have $\Delta E/E \ll 1$. In this case the trajectory of the electron can be treated as a given. If the degree of nonlinearity of the process satisfies $n = \Delta E/\hbar\omega \gg 1$ (ω is the frequency of the external field), the field in which the electron moves can be assumed to be varying adiabatically slowly. In a semiclassical analysis of the problem, we can thus assume that at each time the interaction of the electron with the target depends on the instantaneous value of the energy of the electron in the wave field. Furthermore, the trajectory of an electron undergoes a substantial change in direction over macroscopic distances, no shorter than the wavelength of the external field, λ. Since we have $\lambda \gg (\hbar/m\omega)^{1/2} \gg (\hbar^2/m\Delta E)^{1/2}$, the interaction region characterizing these processes is such that the motion of the electron over this region can be treated as rectilinear.

Under these conditions, the probability per unit time for the occurrence of any process, which is stimulated by the collision of relativistic electrons in the ultrastrong electromagnetic field, can be found from the formula

$$W(t) = j(t)\sigma(t)n_0 , \qquad (1)$$

where $j(t) = v(t)n_e$ is the flux density of the electrons, with a number density n_e and a velocity $v(t)$, $\sigma(t)$ is the cross section for the process, and n_0 is the number density of target particles. The cross section $\sigma(t)$ depends on only the electron energy in the external field at the given time, $E(t)$.

The cross sections for various processes can be found by the equivalent-photon method. Under the conditions of this problem, that statement means that the instantaneous value of the cross section for a process is determined by the spectrum of equivalent photons of the electron with the instantaneous energy $E(t)$. This energy varies adiabatically slowly in time. The equivalent-photon spectrum of an electron is[5]

$$n(\omega)d\omega = \frac{2\alpha}{\pi} \ln(E(t)/\hbar\omega)\frac{d\omega}{\omega} \quad . \tag{2}$$

The cross sections for processes stimulated by electron impact, $d\sigma_e$, can be calculated easily from the corresponding cross sections for processes involving real photons, $d\sigma_{ph}$:

$$d\sigma_e = \int d\omega_1 n(\omega_1) d\sigma_{ph}(\omega, \omega_1). \tag{3}$$

For example, the bremsstrahlung cross section at frequencies corresponding to resonant polarization of the target is

$$\frac{d\sigma_T}{d\omega} = \frac{2\omega^3\alpha}{\Gamma_n} \sum_{\rho=1,2} \int |(\mathbf{d}_{2n}\mathbf{e}'^*_\mu)(\mathbf{d}_{n1}\mathbf{e}_\rho)|^2 do \ln(E(t)/\hbar\omega), \tag{4}$$

where n is the resonant level, Γ_n is its width, $\mathbf{d}_{n1}$ is the dipole matrix element for the transition from the $|1\rangle$ ground state to the $|n\rangle$ resonant state, $\mathbf{d}_{2n}$ is the matrix element for the transition $|n\rangle \to |2\rangle$ (to the final state), $\mathbf{e}_\rho$ is the polarization vector of the equivalent photon, and $\mathbf{e}'_\mu$ is the polarization vector of the bremsstrahlung. In (4) we have taken an average over the polarization directions of the equivalent photons, and we have integrated over all possible polarization directions of the bremsstrahlung photon. In deriving this expression we also used the known expression for the cross section for resonant scattering of photons.[5]

Correspondingly, the cross section for the excitation of the $|2\rangle$ state (without consideration of direct multiphoton excitation) can be found as the cross section for the absorption of equivalent photons with a frequency near the transition frequency ω_{21}:

$$\sigma_{21} = \frac{64\pi^2\alpha}{3} |\mathbf{d}_{21}|^2 \ln(E(t)/\hbar\omega_{21}). \tag{5}$$

The behavior of the electron energy as a function of the frequency and field strength of a plane electromagnetic wave of arbitrary polarization is known.[6] For an electron which is on the average at rest, i.e., whose energy is determined completely by the external field at the given instant, the energy is given by

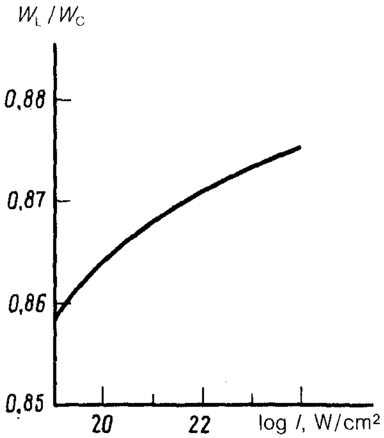

FIG. 1. Ratio of the probability for the excitation of the resonant transition of the hydrogen-like Ca ion in a collision with electrons in linearly and circularly polarized optical fields from a neodymium laser, versus the light intensity. Both probabilities have been averaged over the period.

$$E = c\gamma + \frac{e^2}{2\gamma c}(\mathbf{a}^2 - \overline{\mathbf{a}^2}), \tag{6}$$

$$\gamma^2 = m^2c^2 + \frac{e^2}{c^2}\overline{\mathbf{a}^2},$$

where **a** is the vector potential of the wave field, which depends on the quantity $\xi = ct - x$ as the wave propagates along the x axis, and $\overline{\mathbf{a}^2}$ is the expectation value of the square of the vector potential over the oscillation period. The probabilities for the processes depend on the polarization of the external field. Figure 1 shows the ratio of the probability per unit time for the excitation of the resonant transition for the hydrogen-like Ca ion with a linear polarization (W_L), averaged over the period, to the corresponding value in the case of circular polarization (W_C), as a function of the intensity of the external field, from a neodymium laser ($\omega = 9440\ \text{cm}^{-1}$).

The approach proposed in this letter might be used in problems involving the production of γ rays and electron-positron pairs, and also in the problem of electrostimulated nuclear reactions.

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