# Energy spectrum of 2D electrons in inclined magnetic field

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A $2d$ electron gas in a AlGaAs–GaAs heterojunction is used to show that in strong inclined magnetic fields $H$, when $\hbar\omega_c = \hbar cH/mc$ is much greater than the quantum-size energy, the splitting does not depend on the magnetic field and is determined by the shape of the potential well and by the slope of the field.

**1.** The inclined-field method has been used extensively in experiments with $2d$ electrons in order to demonstrate their two-dimensional nature,[1] to study the spin splitting,[2] and for other purposes. It was generally assumed that the energy level of the quantum states is determined exclusively by the normal component of the magnetic field.[3] The effect of the parallel component of the magnetic field $H_{\parallel}$ on the splitting of the Landau levels was studied by the cyclotron-resonance methods[4] under conditions where $H_{\parallel}$ pushes apart the $n$th and $n+1$ Landau levels from different quantum-size subbands. In weak magnetic fields in which the magnetic length $\lambda_{\parallel} = \lambda_H/\sin^{1/2}\alpha$, determined from the longitudinal component of the magnetic field $H_{\parallel} = H\sin\alpha$, is much greater than the width of the well $\lambda_z$, its effect on the spectrum of the $2d$ carriers should be taken into account in perturbation theory. Disregarding the possible changes in the form of the quantizing potential, we can assume that (a) in low fields the correction $\delta E_n$ to the energy $E_n$ of the $n$th quantum-size level arises in the second order in $H_{\parallel}$, $\delta E_n \sim (\hbar\omega_c \sin\alpha)^2/(E_1 - E_0)$, and the position of the Landau levels changes solely due to the parallel-field-induced anisotropy of the effective mass of a $2d$ carrier, and (b) upon crossing the adjacent Landau levels from different subbands—the ground subband and the first excited subband, for example—the magnetic-field component parallel to the layer lifts the degeneracy and causes the levels to push each other apart

$$\Delta E^{n,\,n-1}_{0,1} = \frac{|\langle 0|z|1\rangle|}{\lambda_H \cos^{1/2}\alpha}\,(2n)^{1/2}\,\hbar\omega_c \sin\alpha .$$

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The crossing of the $n$th and $n-k$th Landau levels, $k>1$, is not accompanied by their splitting, at least within corrections quadratic in $H_{\parallel}$. The cyclotron rotation has, on the whole, a purely two-dimensional nature and is determined solely by the normal component of the field, $\omega_c = eH\cos\alpha/mc$.

2. Of interest, in our view, is the other limit, where $\lambda_{\parallel} \ll \lambda_z$, and the fastest motion is the cyclotron rotation of electrons around the tilted magnetic field of frequency $\omega_c = eH/mc$, which is determined by its total value. In other words, a strong longitudinal field overcomes the size quantization in a layer and effectively trimerizes the particle motion, which now corresponds to a classical motion along a cycloid with the axis directed along the field. Under the conditions described above, the ground-state energy therefore shifts faster with increasing field than it does in low fields. The motion along the magnetic field can be viewed as an adiabatic motion, and the low-lying part of the carrier spectrum is determined, in the limit $H\to\infty$, by the position of the levels in the one-dimensional potential

$$U_\alpha(\zeta) = U_0(\zeta\cos\alpha), \qquad \hat{H} = p_\zeta^2/2m + U_\alpha(\zeta) \tag{1}$$

whose scale transformation differs from that of the initial transformation. (Here $\zeta$ denotes the electron coordinate along the magnetic field.) The splitting of the low-lying levels $E^*_{nm}(\alpha)$ therefore does not depend on the magnetic field in this limit and is much smaller than the intersubband splitting $E_{nm}$. For a model-based single-parameter power potential $U(r)\sim z^\nu$, for example, we have

$$E^*_{nm}(\alpha) = \cos^{2\nu/(\nu+2)}(\alpha)E_{nm}, \tag{2}$$

and for an exactly solvable problem of a parabolic well[5,6] we have

$$E^*_{nm}(\alpha) = (n-m)\{(\omega_c^2 + E_{10}^2)/2$$
$$-[(\omega_c^2 + E_{10}^2)^2/4 - \omega_c^2E_{10}^2\cos^2\alpha]^{1/2}\}^{1/2} \sim \cos\alpha E_{nm}.$$

The capacity of each level, on the other hand, is determined exclusively by a normal component of the magnetic field.

3. We have studied the energy spectrum of electrons in an inclined field by a method based on the determination of the radiative recombination of $2d$ electrons with photoexcited holes which are bound on acceptors[7–9] in the $\delta$ doped layer. This method can be used to directly measure the energy splitting of the quantum levels. Since there is no depleted layer in the case of photoexcitation,[10] the potential well near the interface is broad enough, and the quantization energy decreases. The number of states which manifest themselves in the luminescence spectra is determined by the filling which is controlled by pumping.

Figure 1 shows the emission spectra measured in a single AlGaAs–GaAs heterojunction with $n_s = 3\times10^{11}$ cm$^{-2}$ in perpendicular and tilted magnetic fields. In all three cases the normal component of the magnetic field is the same, $H_\perp = 2.2$ T (which corresponds to a filling factor $\nu = 6$). From these spectra we can draw the following conclusions. (a) At $H_\perp = 2.2$ T and $\alpha < 68°$ the splitting of the Landau levels

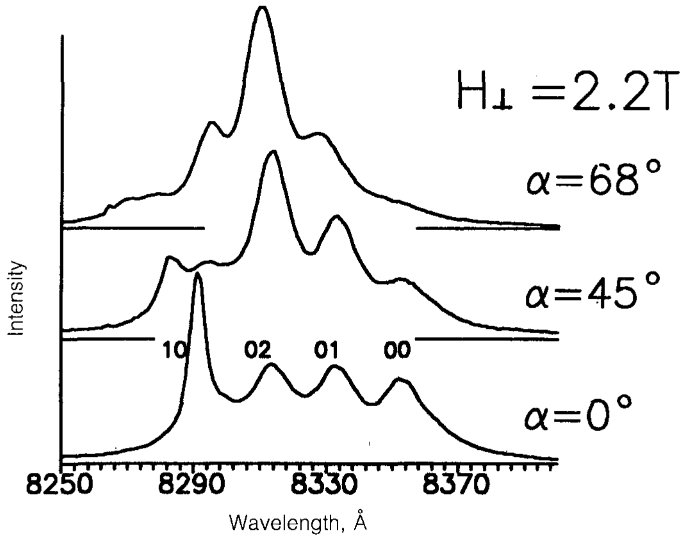


FIG. 1. Spectra of the radiative recombination of 2*d* electrons with holes bound at the $\delta$ layer of the acceptors. $\alpha$ is the slope of the magnetic field with respect to the normal to the 2*d* layer; $n_s = 3\times 10^{11}$ cm$^{-2}$.

is determined by the normal component $H$ and is therefore approximately the same in all three cases. (b) A comparison of the intensities of the recombination lines of electrons from the lowest subband shows that the $z$ dependences of the electron wave functions of the various Landau levels of this subband, which are the same in the perpendicular magnetic field, differ in the tilted field because of the admixing of the wave functions of higher-lying levels. This difference is most pronounced in strong fields which give rise to the trimerization of the cyclotron rotation described above, and the amplitude of the wave functions, which is situated a distance $z_h$ from the interface at the place where the holes are localized, is determined by the extent of the wave functions of the bound states in the one-dimensional potential (1). A calculation has shown that a relative intensity of the $n$th line of the parabolic well, normalized to the ground state, increases in a power-law fashion with increasing number of the level

$$I_n / I_0 \sim \{2z_h^2 \cos\alpha / \lambda_z^2\}^n n! , \qquad \lambda_z \sim \hbar/(mE_{10})^{1/2} .$$

(c) Figure 1 shows that the rate at which the electrons of the excited quantum-size subband recombine decreases appreciably in an inclined magnetic field, in keeping with the expected contraction of the wave function in these states.

Figure 2 shows the position of the energy levels versus the magnetic field and the level splitting, measured from the luminescence spectra at $n_s = 3\times 10^{11}$ cm$^{-2}$ for 0° and 45° slopes. We see from Fig. 2 that in an inclined magnetic field the experimental dependence begins to deviate from a linear behavior at large values of $H$ and the splitting of the adjacent levels no longer depends on the magnetic field. Furthermore, with increasing slope, the residual splitting decreases. Figure 2c shows the splitting of the zero level and the first Landau level and that of the first and second levels as a function of the magnetic field, measured at $\alpha = 45°$ and $n_s = 3\times 10^{11}$ cm$^{-2}$. We see

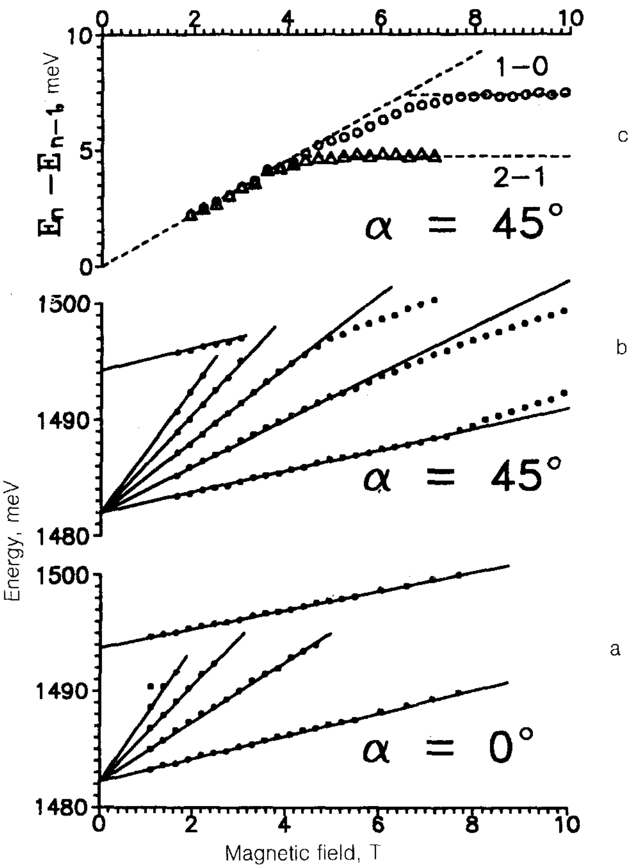


FIG. 2. Position of the lines in the luminescence spectra versus the total field $H$; $n_s = 3\times 10^{11}$ cm$^{-2}$; a) $\alpha = 0$; b) $\alpha = 45°$; c) level splitting versus the total field for $\alpha = 45°$. $H_1^* \sim 4$ T and $H_2^* \sim 7$ T.

that beginning with a certain field $H_n^*$, the splitting becomes virtually independent of $H$. We can assert, therefore, that at $H > H_n^*$ the level splitting is determined by the size quantization of electron motion in a potential well along the direction of the magnetic field. We will assume that it is equal to $E_{nm}^*(\alpha)$. We emphasize that at a fixed concentration $H_n^*$ is virtually independent of the slope, and at a fixed angle $H_n^*$ decreases considerably upon a decrease in $n_s$.

**4**. Information on the slope of the quantum well can be obtained from the plots of $E_{10}^*(\alpha)/E_{10}$ and $E_{nm}^*(\alpha)/E_{10}^*$ vs $\alpha$, which are shown in Fig. 3. The angular dependence of $E_{10}^*/E_{10}$ (Fig. 3a) was determined at various electron densities in the $2d$ channel and was found to be the same for all of the $n_s$. This suggests that the well can be approximated with good accuracy by a power-law dependence in such a way that the $2d$ concentration of carriers will affect only the potential depth. A similar factorization of the concentration and angular dependences of all the splittings in the spectrum follows from the fact that $E_{nm}^*(\alpha)/E_{10}^*(\alpha)$ in Fig. 3b does not depend on the angle $\alpha$. It can be determined from these values that the shape of the well in our case

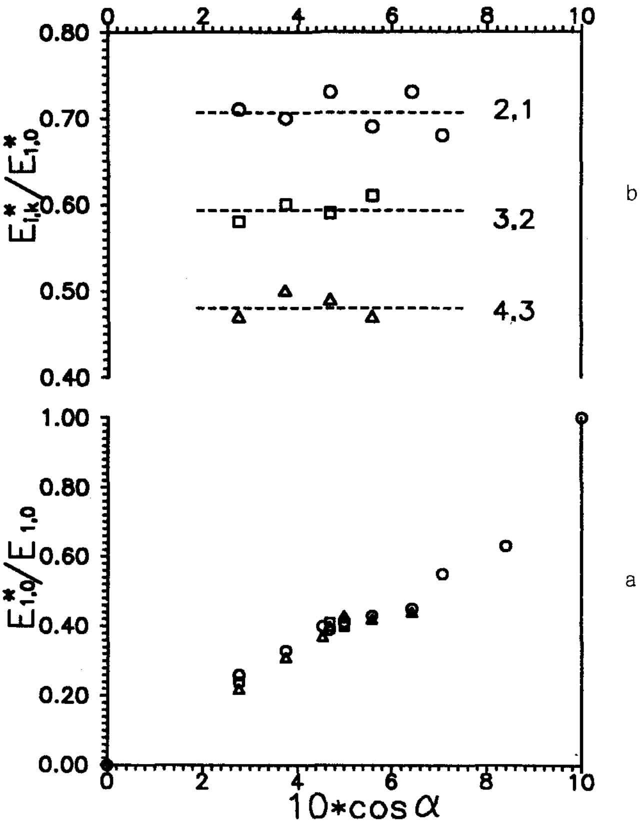


FIG. 3. a) Splitting in the spectrum in strong inclined magnetic fields $E^*_{10} = E^*_1 - E^*_0$, normalized to the intersubband splitting in the absence of a magnetic field versus the angle $\alpha$. 0—$n_s = 3\times10^{11}$ cm$^{-2}$; □—$n_s = 2.2\times10^{11}$ cm$^{-2}$; Δ—$n_s = 1.5\times10^{11}$ cm$^{-2}$; b) $E^*_{nm}(\alpha)/E^*_{10}(\alpha)$ versus the angle $\alpha$ for $n_s = 3\times10^{11}$ cm$^{-2}$.

corresponds to a power-law distribution of the potential $U(z) \sim z^\nu$ with $\nu \approx 0.5$.

The magnetospectroscopy of $2d$ electrons in an inclined field is thus an effective tool for studying the shape of single potential wells that hold electrons.

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