Defects at interface between *A* and *B* phases of superfluid ^{3}He

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Topologically stable structural defects at the interface between the *A* and *B* phases of superfluid ^{3}He are classified. Among these defects there are vortices with ends and boojums.

The interface between the *A* and *B* superfluid phases of liquid ^{3}He is a $2D$ entity which is presently being studied experimentally. The topological defects which may exist at an interface were studied in Ref. 1. The most interesting of these defects are monopole-like defects consisting of vortex lines which have an end point at the *AB* interface. In the present letter we classify defects on the basis of the particular structure of the *AB* interface.

The order parameter in superfluid ^{3}He is the 3×3 matrix $A_{\alpha i}$. In the equilibrium state of the *A* phase, $A_{\alpha i}$ is expressed in terms of the unit vectors $\mathbf{d}$, $\mathbf{e}_1$, and $\mathbf{e}_2$: $A^A_{\alpha i} = \Delta_A d_\alpha (e_{1i} + ie_{2i})$, where $\mathbf{e}_1 \cdot \mathbf{e}_2 = 0$, and the vector $\mathbf{1} = [\mathbf{e}_1, \mathbf{e}_2]$ gives the direction of the orbital angular momentum of the Cooper pairs. In the equilibrium state of the *B* phase, $A_{\alpha i}$ is expressed in terms of the $3D$ rotation matrix R: $A^B_{\alpha i} = \Delta_B e^{i\Phi} R_{\alpha i}$, where Δ_B and Φ are the amplitude and phase of the order parameter of the *B* phase. The interface at which these two phases are linked creates reciprocal boundary conditions, which link the degeneracy parameters of the *A* phase ($\mathbf{d}$, $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{1}$) with those of the *B* phase (Φ and R). These boundary conditions depend on the structure of the interface. Various *AB* interfaces with various types of symmetry were studied in Ref. 2. Near T_c, the most stable *AB* interface has the following asymptotic expressions for the order parameter on the two sides of the interface:[2,3]

$$A^0_{\alpha i}(x = -\infty) = \Delta_A \hat{x}_\alpha (\hat{x}_i - i\hat{z}_i), \qquad A^0_{\alpha i}(x = +\infty) = \Delta_B \delta_{\alpha i}\,, \tag{1}$$

where x runs along the normal to the interface. This is only one of the degenerate states of the interface; others are found by using the symmetry operations which form the group:

$$G = U(1) \times SO(2)^L \times SO(3)^S\,. \tag{2}$$

Here $SO(3)^S$ is the spin-rotation group, which acts on the Greek index of the order parameter, $SO(2)^L$ is the group of orbital rotations around the normal $\hat{x}$, which act on the Latin index of the order parameter, and $U(1)$ is the group of gauge transformations, which alter the phase of the order parameter. As a result, the general expression for the order parameter far from the *AB* interface is

$$A_{\alpha i} = e^{i\Phi} R^S_{\alpha\beta} R^L_{ik} A^0_{\beta k}\,, \tag{3}$$

where R^S is the matrix of $3D$ spin rotations, and R^L is the matrix of orbital rotations around x. It can be seen from (1) and (3) that the asymptotic expressions for the phases are not arbitrary. The orientation of the spin vector **d** of the A phase is related to the orientation of the matrix R of the B phase, since we have $\mathbf{d} = R^S\hat{x}$, and $R = R^S(R^L)^{-1}$. The orbital vector **1** of the A phase is oriented parallel to the plane of the interface, since $\mathbf{1} = R^L\hat{y}$, but the direction of **1** in the plane of the interface is not fixed by the orientation of the matrix R in the B phase, since the orbital rotation of the order parameter of the B phase can be offset by an opposite rotation of the spin space.

Equation (3) does not contain equivalent states, since each element of group G changes the state of the A phase, the state of the B phase, or the states of both phases. This result means that the space of degenerate states of the AB interface coincides with the 5D space of group G; i.e., $R_{AB} = G$. The fundamental homotopy group is

$$\pi_1(R_{AB}) = Z \times Z \times Z_2, \tag{4}$$

so point defects at the AB interface are described by the three integers $\mathbf{N} = (N_1, N_2, N_3)$. Here N_1 is the number of quanta of circulation of the superfluid velocity along a contour on the AB interface which loops a defect; N_2 is the number of rotations of the vector **1** as the defect is looped along the AB interface; and N_3, which is from the group z_2 consisting of the two elements 0, 1, describes the singularities in the field of the matrix R. We now need to determine how the defects at the interface are related to defects in the interior and whether they are end points of singular lines coming out of the volume or isolated point defects on the AB interface. To resolve these questions, we can use the method of relative homotopy groups, which have been used to classify defects at the surface of an ordered medium. It follows from this analysis that a defect with an odd value of $N_1 + N_2$ necessarily propagates into the A phase, while one with an even value of $N_1 + N_2$ has no singularities in the A phase. A defect with a nonzero N_1 is an end point of a B-phase vortex with N_1 circulation quanta, while a defect with $N_3 = 1$ is an end point of a disclination in the B phase. The quantity N_2 has no effect at all on the behavior of the order parameter in the B phase. As a result, the defects can be grouped in five categories.

I. The end points of line singularities of the A phase without a singularity in the B phase. These are defects with topological charges $\mathbf{N} = (0.2k + 1.0)$. The elementary defect $\mathbf{N} = (0, 1, 0)$ is a disgyration in the field of the vector **1** which terminates at the AB interface [Fig. 1(a)] or an end point of a singular vortex [Fig. 1(b)], depending on which of the line defects of the A phase has the lower energy.

II. The end points of line singularities of the B phase without a singularity in the A phase. Here there are two subtypes. IIa. The elementary representatives of this subtype are defects with $\mathbf{N} = (\pm 1, \pm 1.0)$. These are the end points of B-phase vortices with $N_1 = \pm 1$ circulation quanta. The end points are point defects in the field of the vector **1** of the A phase, i.e., boojums [Fig. 2(a)]. A boojum is characterized by two quantum numbers. One is the number of rotations of the vector **1** over the interface, N_2. the second is the degree of the mapping of the hemisphere around the boojum from the A-phase side onto the sphere of the vector **1**. It can be expressed in terms of an integral in the yz plane which passes parallel to the AB interface in the A phase: $N = 1/4\pi \int dydz\mathbf{1}[\partial_y\mathbf{1}, \partial_z\mathbf{1}]$. According to the Mermin-Ho relation, it is relat-

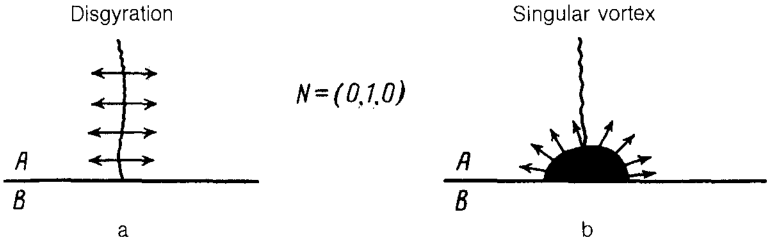

FIG. 1. a—Disgyration in the field of the vector **l** (shown by the arrows); b—singular vortex in the *A* phase. Each terminates at the interface between the *A* and *B* phases at a point at which the vector **l** rotates: $N_2 = 1$. The two defects belongs to the same topological class.

ed to the number of circulation quanta in the vortex which arrives at this point from the *B* phase: $N = (1/2)N_1$. IIb. These are defects with $\mathbf{N} = (0, 0, 1)$; they are end points of a *B*-phase disclination. In contrast with vortices, the disappearance of a disclination at the interface does not require the existence of boojums in the *A* phase [Fig. 2(b)]. The other type-II defects are combinations of the elementary defects [see, for example, Fig. 2(c)].

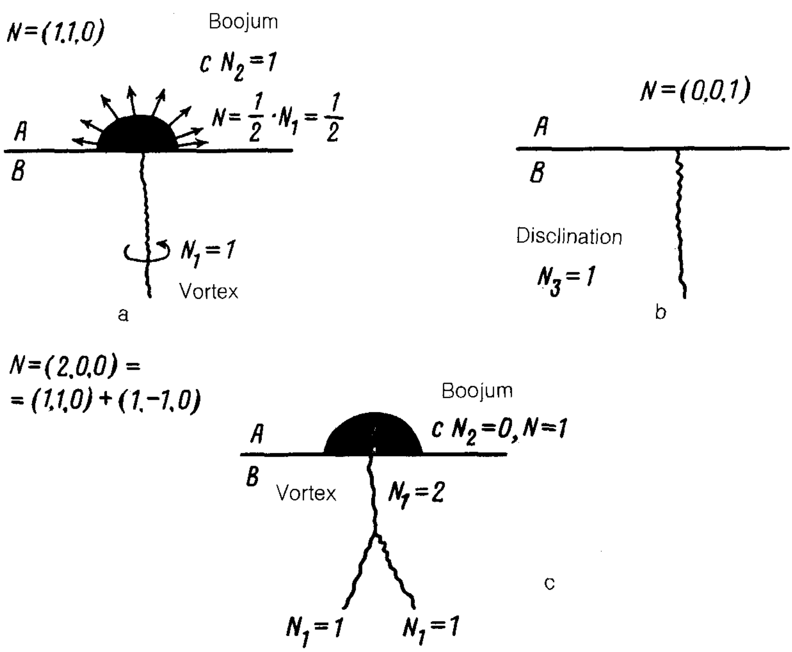

FIG. 2. a,c—Quantized vortices with (a) one and (c) two circulation quanta in the *B* phase terminate at *A*-phase boojums; b—disclination of the *B* phase terminates at the *AB* interface without the formation of a boojum in the *A* phase.

III. Points at which line defects intersect the AB interface. An elementary defect of this type is a single-quantum vortex which intersects the AB interface: $\mathbf{N} = (1, 0, 0)$ [Fig. 3(a)]. It can be thought of as a combination of the defects in Figs. 1(b) and 2(a): $(1, 0, 0) = (0, 1, 0) + (1, -1.0)$ [the defect in Fig. 2(a) must be taken with the opposite $N_2 = -1$].

IV. Isolated point defects at the AB interface: AB boojums. The elementary defect of this type, $\mathbf{N} = (0, 2, 0)$ [Fig. 3(b)], can be generated by combining two of the defects in Fig. 1(b): $(0, 2, 0) = (0, 1, 0) + (0, 1, 0)$. Here we are making use of the known fact that a connection of two singular lines in the A phase leads to a nonsingu-

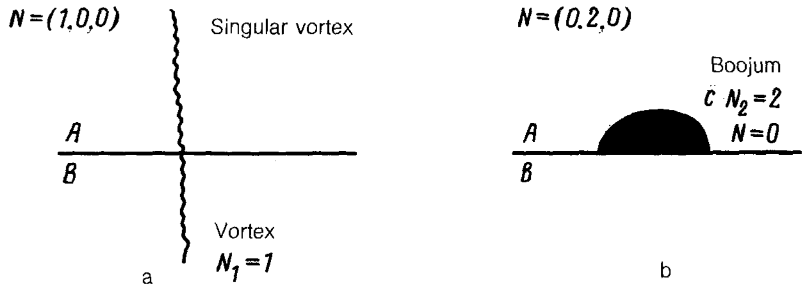

FIG. 3. a—Quantized vortex which intersects the AB interfaces without the formation of boojums; b—isolated boojum without singularities in the volume

lar state. According to the Poincaré theorem regarding the index of a vector field (1) tangent to a closed surface, a boojum of this sort should exist at the surface of a droplet of a droplet of the A phase in the B phase or one of the B phases in the A phase.

This classification of defects is determined by the particular structure of the order parameter in the AB interface, which dictates the reciprocal boundary conditions for the degeneracy parameters of the A and B phases. For interfaces of other types,[2,5] the classification will be different.

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