Impurity-stimulated superconductivity

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Inelastic scattering by impurities can raise the superconducting transition temperature. An explanation is proposed for the nonmonotonic $T_c(x)$ dependence in the heavy-fermion compound $U_{1-x}Th_xBe_{13}$.

Nonmagnetic impurities at a sufficiently low concentration do not affect the temperature of the transition to a superconducting state with s pairing, but they do suppress any other type of superconductivity (p, d, f), so that T_c decreases in proportion to the impurity concentration. Magnetic impurities also suppress a superconductivity with s pairing. This situation is characteristic of elastic scattering by impurities, i.e., of the case in which the interaction of the conduction electrons with the impurities does not include a change in the electron energy or of a transition of impurities to excited states. As well will show below, the incorporation of these effects may substantially raise the superconducting transition temperature for a pairing of any type. The simplest model for which analytic calculations are possible is that in which the impurities are two-level centers with a splitting energy E distributed over an interval $\sim E_0$, with a normalized density $W(E)$, in a superconductor with s pairing. This model has been studied by Élashberg[1] with reference to the renormalization of the spectrum of a normal metal.

As in the ordinary impurity technique,[2] we write the expressions for the normal Green's function $G(\omega_n, \mathbf{p})$ and the superconducting Green's function $F^+(\omega_n, \mathbf{p})$ as follows:

$$G(\omega_n, \mathbf{p}) = -\frac{i\omega_n - \overline{G} + \xi}{-(i\omega_n - \overline{G})^2 + \xi^2 + (\Delta + \overline{F}^+)^2}$$

$$F^+(\omega_n, \mathbf{p}) = \frac{\Delta + \overline{F}^+}{-(i\omega_n - \overline{G})^2 + \xi^2 + (\Delta + \overline{F}^+)^2} \qquad (1)$$

Here $\overline{G}$ and $\overline{F}^+$ are the corresponding eigenenergy parts

$$\overline{G} = n_i T \sum_{n'} \int \frac{d^3p'}{(2\pi)^3} |V_{pp'}|^2 \int dE W(E) \frac{2E \tanh(E/2T)}{(\omega_n - \omega_n')^2 + E^2} (\omega_{n'}, \mathbf{p}')$$

$$\overline{F}^+ = n_i T \sum_{n'} \int \frac{d^3p'}{(2\pi)^3} |V_{pp'}|^2 \int dE W(E) \frac{2E\, \mathrm{th}(E/2T)}{(\omega_n - \omega_{n'})^2 + E^2} F^+(\omega_{n'}, \mathbf{p}'), \qquad (2)$$

$\omega_n = (2n+1)\pi T$, n_i is the impurity concentration, and Δ satisfies the self-consistency equation

$$\Delta = gT \sum_n \int \frac{d^3p}{(2\pi)^3} F^+(\omega_n, \mathbf{p}), \tag{3}$$

where g is an interaction constant.

Introducing

$$\begin{aligned} i\omega - \overline{G} &= i\omega\eta(\omega) \\ \Delta + \overline{F}^+ &= \Delta\zeta(\omega) \end{aligned} \tag{4}$$

and switching to an integration over the Fermi surface (dS and $d\xi$), we find from system (1), (2)

$$\begin{aligned} \eta &= 1 + \frac{T}{2\pi\tau\omega_n} \sum_{n'} \int d\xi \int dE \, \frac{W(E)2E\tanh(E/2T)}{(\omega_n - \omega'_n)^2 + E^2} \, \frac{\omega_{n'}\eta}{\xi^2 + \omega_{n'}^2\eta^2 + \Delta^2\zeta^2} \\ \zeta &= 1 + \frac{T}{2\pi\tau} \sum_{n'} \int d\xi \int dE \, \frac{W(E)2E\tanh(E/2T)}{(\omega_n - \omega'_n)^2 + E^2} \, \frac{\zeta}{\xi^2 + \omega_{n'}^2\eta^2 + \Delta^2\zeta^2} \end{aligned} \tag{5}$$

Here

$$\frac{1}{\tau} = \frac{n_i}{(2\pi)^2} \int \frac{dS}{v} |V_{pp'}|^2 .$$

In the case $T \gg E_0$, the sum is dominated by the term with $n' = n$; hence $\eta = \xi = \text{const}$. Substituting the second equation in (1) into (3) and taking the limit $\Delta \to 0$, we reach the standard conclusion that nonmagnetic impurities do not renormalize the transition temperature. The situation is different at $T \ll E_0$. Introducing the ratio $\alpha = \zeta/\eta$, and taking the limit $\Delta \to 0$, we find from (5)

$$\begin{aligned} &\alpha\left\{1 + \frac{T}{2\tau\omega_n} \sum_{n'} \int dE \, \frac{2EW(E)}{(\omega_n - \omega_{n'})^2 + E^2} \operatorname{sign}\omega'\right\} \\ &= 1 + \frac{T}{2\tau} \sum_{n'} \int dE \, \frac{2EW(E)}{(\omega_n - \omega_{n'})^2} \, \frac{\alpha}{|\omega_{n'}|} . \end{aligned} \tag{6}$$

From (3) and the second equation in (1) we find

$$\pi\rho_0 gT \sum_n \frac{\alpha}{|\omega_n|} = 1, \tag{7}$$

where $\rho_0 = mp_F/2\pi^2$ is the density of states.

In order to solve Eq. (6) analytically, we make the natural assumption that the frequency at which the summation is cut off in (7) is on the order of E_0. In this case we find from (6)

$$\alpha(1+\lambda)=1+\frac{\lambda}{\rho_0 g},$$

where

$$\lambda=\frac{1}{\pi\tau}\int\frac{W(E)dE}{E}\gtrsim\frac{1}{\pi\tau E_0}. \tag{8}$$

We can thus find α; substituting it into (7), we find

$$T_c\approx E_0\exp\{-\frac{1+\lambda}{\rho_0 g+\lambda}\}. \tag{9}$$

In the low-concentration limit we then find

$$T_c=T_{c0}[1+\lambda((\ln\frac{E_0}{T_c})^2-\ln\frac{E_0}{T_c})]. \tag{10}$$

The increase in T_c with the concentration of centers at which inelastic scattering occurs is almost certainly pertinent to the nonmonotonic behavior of T_c in the heavy-fermion compound $U_{1-x}Th_xBe_{13}$. By replacing magnetic uranium atoms, the nonmagnetic thorium atoms cause T_c to decrease as x is increased to a concentration $x_m=1.7\%$. In the interval $1.7\%<x\lesssim 3\%$, this decrease gives way to a sharp increase in T_c (Refs. 3 and 4). Impurities, on the other hand, cause qualitative changes in the temperature dependence of the resistance. An increase in the impurity concentration from 0 to 1.7% is accompanied by a decrease in T_m—the position of the maximum on the temperature dependence of the resistance—which separates the regions of coherent ($T<T_m$) and incoherent ($T>T_m$) scattering.[3,5] In $U_{1-x}Th_xBe_{13}$ we have $T_m(x=0)=2.3$ K. According to the general understanding, the increase in the resistance with decreasing temperature at $T>T_m$ is due to scattering by "nearly" independent Kondo centers.[1)] $T<T_m$, a coherent normal state forms in a periodic lattice of Kondo centers; in this state, the decrease in the resistance with the temperature is due to a scattering of charges by each other. The addition of a thorium impurity lowers the temperature at which the coherent normal state is established, T_m. Note that the increase in T_c with the thorium concentration begins as soon as we reach the concentration at which T_m becomes comparable to T_c; i.e., the increase begins where the material goes into a superconducting state "without having managed" to pass through the "phase" of a coherent normal state (Fig. 1). A natural explanation for this situation can be found by assuming that the nonmagnetic thorium atoms which replace the magnetic uranium atoms act as magnetic impurities, suppressing the superconductivity in the coherent region (i.e., at $x<x_m$), but in the incoherent regime ($x>x_m$)

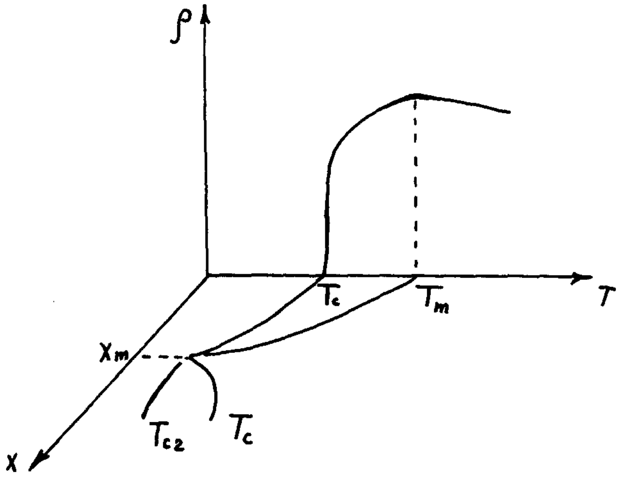

FIG. 1. Schematic phase diagram of $U_{1-x}Th_xBe_{13}$ in the (x,T) plane. T_c—Line of the transition to the superconducting state; T_m—line of the maximum on the plot of the resistivity ρ versus the temperature (this line separates the region of coherent scattering, $T < T_m$, from that of incoherent scattering, $T > T_m$); T_{c2}—line of the additional phase transition inside the superconducting region, at $x > x_m$.

inelastic carrier scattering processes capable of stimulating a superconductivity, as was shown above, come into play.

The nonmonotonic dependence $T_c(x)$ in UBe_{13} is observed only for a thorium impurity. A possible explanation for this observation is that only when uranium is replaced by thorium does the coherent regime (the concentration range with $T_m - T_c > 0$) manage to be replaced by an incoherent regime while the superconductivity is not yet totally suppressed ($T_c > 0$). If T_c vanishes at a certain impurity concentration while the inequality $T_m > 0$ remains in force, as in $U_{1-x}Th_xBe_{13}$ under pressure,[4] a superconductivity may be restored only at even higher concentrations, at which T_m also vanishes. It is possible that a vanishing of T_m occurs for other impurity atoms at comparatively higher concentrations than in the case of thorium, and this is the reason for the restoration of the superconductivity. These arguments could be tested by direct experiment, e.g., with the help of a La impurity, which is also nonmagnetic and which causes approximately the same expansion of the UBe_{13} lattice as a thorium impurity. A combination of La and Th impurities might be used.

In conclusion it is pertinent to recall that in the region $x > x_m$ in the superconducting phase there is a second phase transition (the line T_{c2} in Fig. 1), whose nature has yet to be established. The present status of experimental work pertinent to this transition is described in Ref. 6.

In summary, we would like to repeat the basic assertion of this paper: that inelastic scattering by impurity centers may be the reason for the nonmonotonic $T_c(x)$ dependence, i.e., the concentration dependence of the superconducting transition temperature in $U_{1-x}Th_xBe_{13}$.

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[1)] We are using the term "Kondo center" in a broader meaning than an impurity which is interacting with conduction electrons through an exchange contact interaction. The two-level impurity centers discussed above, for which the inelastic scattering occurs in the charge channel, rather than in the spin channel, might also be categorized as such entities.

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