Residual quantum conductivity under Coulomb-blockade conditions

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A tunnel junction containing a metal grain in a barrier is analyzed. The conductance is calculated as a function of the grain potential in the low-temperature limit.

As was shown in the classic experiments by Zeller and Giaver,[1] metal inclusion (grains) in a barrier determine the conductance of a tunnel junction, since most of the tunnel current flows through such inclusions. A tunneling electron alters the charge of the grain by $\pm e$, and the corresponding change in the energy of the grain is $\Delta E \sim e^2/C$, where C is the capacitance of the grain. The real states at a grain can be used during tunneling by only those electrons whose energies exceed ΔE. The corresponding component of the junction conductance is incorporated in a kinetic equation[2,3] with a temperature dependence of an activation nature, $G \propto \exp(-\Delta E/T)$. The assertion that G vanishes as $T \to 0$ is called the "Coulomb blockade" of one-electron tunneling.[3] In the present letter it is shown that at low temperatures transitions through virtual states at the grain must be taken into account, and because of these transitions, the conductance does not vanish as $T \to 0$.

We describe a tunnel junction containing a grain in a barrier (Fig. 1) by the Hamiltonian

$$H = H_0 + H_T$$

$$H_0 = \sum_k \epsilon_k A_k^+ A_k + \sum_k \epsilon_k B_k^+ B_k + \sum_p \epsilon_p c_p^+ c_p + \frac{\hat{Q}^2}{2C} + \varphi \hat{Q}, \qquad (1)$$

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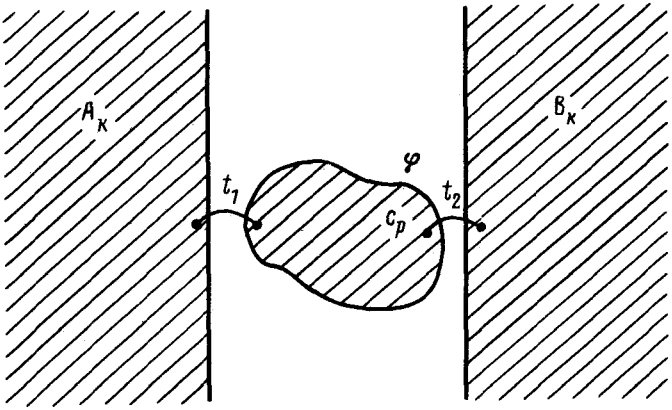

FIG. 1.

$$H_T = \sum_{kp} t_1(A_k^+ c_p + c_p^+ A_k) + \sum_{kp} t_2(B_k^+ c_p + c_p^+ B_k).$$

Here A_k, B_k, and C_p are operators which annihilate an electron in the left and right banks of the junction and at the grain, respectively; ϵ_k and ϵ_p are their energies, reckoned from the Fermi level;

$$\hat{Q} = e \sum_p (C_p^+ C_p - \theta(-\epsilon_p)) \tag{2}$$

is the grain charge operator [$\theta(x)$ is the unit step function]; and t_1 and t_2 are matrix elements which describe tunneling transitions between the banks and the grain. We treat the grain potential φ as an independent parameter, since there is the possibility of varying this potential experimentally[4,5] with the help of an auxiliary electrode (a gate).

The potential shift $\varphi \to \varphi + e/C$ in Hamiltonian (1) is equivalent to a redefinition $\hat{Q} \to \hat{Q} + e$. In other words, according to (2), it is equivalent to a shift of the energies ϵ_p by an amount equal to the spacing of the grain levels, ν_g^{-1} (in such a way that there is one less state under the Fermi level; here ν_g is the density of states in the grain). In a macroscopic grain, the distance between levels is small in comparison with the Coulomb energy e^2/C, and this shift has no effect on the macroscopic properties of the system. The conductance $G(\varphi)$ is therefore periodic in φ with a period e/C (see also Refs. 6 and 7), so we will restrict the discussion below to the interval

$$-\frac{e}{2C} < \varphi < \frac{e}{2C}. \tag{3}$$

We consider only the case of tunnel barriers with a low transmission, which is the case of most interest experimentally.[3] In this case, the conductances

$$G_{1,2} = G_q \nu \nu_q t_{1,2}^2 \tag{4}$$

are small in comparison with the fundamental quantity $G_q = 2\pi e^2/\hbar$ (here ν is the density of states in the banks).

We begin our calculation of the junction conductance $G(\varphi)$ at $T=0$ by finding the probability for the tunneling of an electron from the left bank to the right one in the first nonvanishing order of a perturbation theory in H_T. This probability is determined by the matrix element of the second term, $H_T(E-H_0)^{-1}H_T$, of the series for the $\hat{T}$ operator calculated between the states $A_k^+|\Phi>$ and $B_{k'}^+|\Phi>$ (here $|\Phi>$ is the H_0 ground state, in which all levels below the Fermi level are filled):

$$\langle \Phi | B_{k'} \hat{T} A_k^+ | \Phi \rangle = -\sum_p \frac{t_1 t_2}{\epsilon_p + \left(\frac{e^2}{2C} + e\varphi\right)} \theta(\epsilon_p) + \sum_p \frac{t_1 t_2}{-\epsilon_p + \left(\frac{e^2}{2C} - e\varphi\right)} \theta(-\epsilon_p). \quad (5)$$

The two terms in (5) correspond to virtual states in which there is respectively an additional electron or hole in the grain. Substituting (5) into the known relation

$$G = G_q \sum_{kk'} |\langle \Phi | B_{k'} \hat{T} A_k^+ | \Phi \rangle|^2 \delta(\epsilon_k) \delta(\epsilon_{k'}), \quad (6)$$

we find, in lowest-order perturbation theory,

$$G(\varphi) = \frac{G_1 G_2}{G_q} \ln^2 \frac{\frac{e}{2C} + \varphi}{\frac{e}{2C} - \varphi}. \quad (7)$$

Over most of the interval in (3), the temperature-independent conductance which we have found, (7), is greater than the activation value[6] derived by the kinetic-equation method at temperatures

$$T \ll \frac{e^2}{2C \ln \frac{G_q}{G_1 + G_2}}.$$

As we will see below [see (8)], expression (7) gives a correct description of the behavior $G(\varphi)$ over nearly the entire interval in (3). At $\varphi = \pm e/2C$, however, the expression for the states of the grain with different charges[2,6] leads to a singularity in (7). The calculation of G near the ends of interval (3) thus requires dealing with higher orders of perturbation theory.

Since it is not possible to sum the entire perturbation-theory series, we will calculate $G(\varphi)$ in the leading-log approximation. This calculation can be carried out by the renormalization-group method which Anderson has applied[8] to the Kondo problem. In this method, we systematically reduce the band width μ and add to the Hamiltonian terms such that the amplitude for the transition $k \to k'$ for the Fermi operators does not change. To take this renormalization into account in second-order perturbation theory corresponds to adopting the leading-log approximation. Carrying out this procedure, we can generalize (7) to the region of φ values near $\pm e/2C$:

$$G(\varphi) = \frac{G_1 G_2}{G_1 + G_2} \frac{1}{4} \tan^2 \left\{ 2\sqrt{\frac{G_1 + G_2}{G_q}} \ln \frac{e/2C + \varphi}{e/2C - \varphi} \right\}. \quad (8)$$

It can be seen from (8) that the leading-log approximation cannot be used to study the narrow neighborhoods $|\varphi \pm e/2C| \lesssim U^*$, where

$$U^* = \frac{e}{C} \exp\{-\frac{\pi}{4}\sqrt{\frac{G_q}{G_1+G_2}}\}. \tag{9}$$

At the boundary of the region of applicability of (8), the conductance reaches a value

$$G \sim G_q \frac{G_1 G_2}{(G_1+G_2)^2}. \tag{10}$$

As φ moves closer to the charge-degeneracy points $\pm e/2C$, there is no change in estimate (10).

In our original Hamiltonian, (1), the electrons were assumed spinless. Spin can be dealt with through a redefinition: $G_q = 4\pi e^2/\hbar$.

At a nonzero temperature or if a voltage is applied, inelastic-tunneling processes will contribute to the current.[9]

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