Estimate of *t*-quark mass from the decay $K_L^0 \to \mu^+\mu^-$

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The branching ratio for the decay $K_L^0 \to \mu^+\mu^-$ is calculated on the basis of a quark model with QCD corrections. A comparison of the result with experiment yields an estimate of the mass of the t quark: $m_t \approx 150 \pm 35 \pm 15 \pm 7\ \mathrm{GeV/c^2}$.

In several recent studies, the contribution of the t quark to certain processes has been examined,[1,2] and the results have been used to estimate the mass of this quark. The situation which has developed is reminiscent of the corresponding situation just before the discovery of charm, when the theoretical work pointed out the necessary existence of a C quark with a mass $\sim$1–2 GeV/c^2. In the present letter we estimate the mass of a t quark by analyzing its contribution to the $K_L^0 \to \mu^+\mu^-$ branching ratio. The amplitude for this process is the sum of a single-loop component ($1L$) and a two-loop component ($2L$); that these components are comparable was first pointed out in Ref. 3. The first of them describes a decay involving intermediate W and Z bosons. This component is important at distances $\sim 1/m_W$, at which the t-quark component is dominant, while the QCD corrections are small. It is thus calculated in the electroweak theory with free quarks.[4] The amplitude of the ($1L$) component can be put in the form

$$M_{1L} = -N/\sin^2\theta_W \left[\frac{m_c^2}{m_W^2} + \frac{m_t^2}{m_W^2} \mathcal{F}\left(\frac{m_t^2}{m_W^2}\right) \mathrm{Re}\,(V_{td}^* V_{ts}) / \mathrm{Re}\,(V_{cd}^* V_{cs})\right],$$

$$N = \sqrt{2} \times G_F / \sqrt{2} \times \frac{\alpha}{2\pi} \times F_K \times m_\mu \times \sin\theta_c \times \cos\theta_c (\bar{u}(p_1)\gamma_5 u(-p_2)), \qquad (1)$$

$$\mathcal{F}(z) = \frac{1}{4}\left[\frac{4-z}{1-z} + \frac{3z}{(1-z)^2} \times \ln z\right]. \qquad (2)$$

Here m_μ, m_W, m_c, and m_t are the masses of the μ meson, the W boson, and the c and t quarks, respectively; V_{ij} is the Kobayashi-Maskawa mixing matrix; F_K is the form factor of the K meson; θ_c is the Cabibbo angle; and p_1 and p_2 are the 4-momenta of the μ^- and μ^+, respectively.

The two-loop component of the total amplitude is represented by the diagrams in Figs. 1 and 2. This component is important both at intermediate range ($\sim 1/m_c$) and at larger range ($\gtrsim 1/m_k$). It is of fundamental importance to make systematic use of the QCD corrections in the calculation of this component. We use the procedure of Refs. 5 and 6 to calculate the QCD corrections. For the amplitude $M_{2L}^{(1)}$ corresponding to the diagram in Fig. 1 we derive the following expression:

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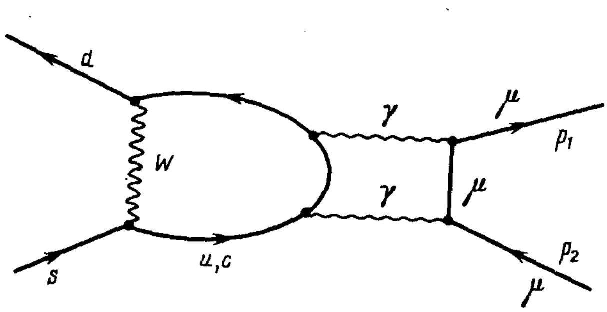

FIG. 1. Two-loop component of the decay $K_L^0 \to \gamma\gamma \to \mu^+\mu^-$, described by the amplitude $M_{2L}^{(1)}$.

$$M_{2L}^{(1)} = \frac{8}{9} \times \frac{\alpha}{2\pi} \times N \times [\eta_1(\mu_0)\, 6\ln\frac{m_c}{\mu_0} + \eta_{LD}(6\ln\frac{\mu_0}{m_\mu} - 2(\ln\frac{m_k}{m_\mu})^2 + \frac{17}{2} - \frac{\pi^2}{6} + i2\pi\ln\frac{m_k}{m_\mu})] . \quad (3)$$

Here μ_0 is a typical hadronic scale, which separates intermediate and large ranges, and $\eta_1(\mu_0)$ is the QCD correction factor for intermediate range scales, given by

$$\eta_1(\mu_0) = \frac{1}{\ln(m_c/\mu_0)} \int_{\mu_0}^{m_c} \frac{d\mu}{\mu} C(\mu) , \quad (4)$$

where $C(\mu)$ is the QCD correction factor for the effective vertex in Fig. 3. We determine the long-range correction factor η_{LD} phenomenologically, through a comparison of the theoretical value for the branching ratio for $K_L^0 \to \gamma\gamma$ with the experimental value. For the amplitude $M_{2L}^{(2)}$ corresponding to Fig. 2, we find a rather lengthy expression. In its place we report here only a numerical result for $\Lambda_{QCD} = 70$ MeV and $\mu_0 = 250$ MeV. For these parameter values we find $\eta_{LD} \approx -1.9$.

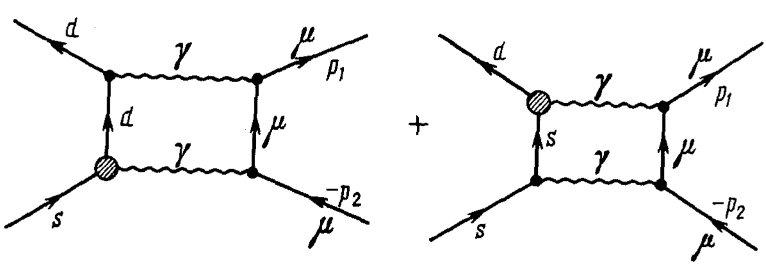

FIG. 2. Two-loop component of the decay $K_L^0 \to \gamma\gamma \to \mu^+\mu^-$, described by the amplitude $M_{2L}^{(1)}$. The hatched region shows the effective vertex corresponding to the emission of a virtual γ ray accompanied by a change in flavor.

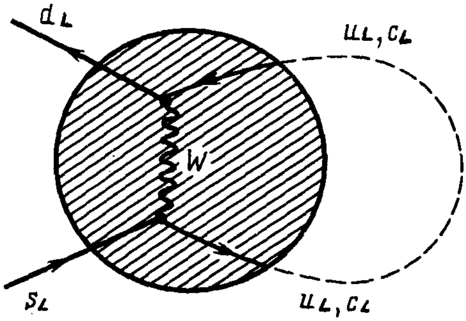

FIG. 3. The hatching shows the effective vertex for which the QCD factor $C(\mu)$ is calculated.

$$\eta_1(250) \approx -\ 0.123;$$

$$M^{(2)}_{2L} \approx +\frac{8}{9}\frac{\alpha}{2\pi} N\ \{10.0 + i\ 0\}\ .$$

An important cancellation occurs in the real part in the two-loop amplitude. As a result, we find

$$M_{2L} \approx -\frac{8}{9}\frac{\alpha}{2\pi}\ N\ \{5.1 + i\ 38.9\}\,. \qquad (5)$$

This assertion remains qualitatively valid over the plausible ranges of $\Lambda_{\rm QCD}$ and μ_0. In the real part of the total amplitude for the decay, the t-quark component is therefore predominant. Summing (1) and (5), we find the following expression for the total amplitude for the decay $K^0_L \to \mu^+\mu^-$:

$$M^{K^0_L\to\mu^+\mu^-} = -N\{\frac{8}{9}\frac{\alpha}{2\pi}(5.1 + i38.9) + \frac{m_c^2}{m_W^2}\ \frac{1}{\sin^2\theta_W} + \kappa z\ \mathcal{F}(z)\},$$

$$\kappa = \frac{\sin^2\gamma(1 - |R|\cos\delta)}{\sin^2\theta_W}\,, \qquad z = \frac{m_t^2}{m_W^2}\ . \qquad (6)$$

Using the experimental data of the ARGUS group for the Kobayashi-Maskawa matrix elements in the Maiani parametrization, and also using data on the width of the decay $K^0_L \to \mu^+\mu^-$, we find

$$m_t \approx 150 \pm 35 \pm 15 \pm 7\ ({\rm GeV/c}^2).$$

The first $\pm$ in this result specifies the error in the measurement of ${\rm Br}(K^0_L \to \mu^+\mu^-)$, the second that in the measurement of $\sin^2\gamma$, and the third that in the measurement of $|R|$. It thus appears to us that it would be of considerable interest to see more-accurate measurements of ${\rm Br}(K^0_L \to \mu^+\mu^-)$ and ${\rm Br}(K^0_L \to \gamma\gamma)$.

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