Multiquantum vortices in ^{3}He-B near the phase transition to ^{3}He-A

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Quantized vortices in ^{3}He-B with a large number of circulation quanta, $N \gg 1$, are locally stable near the temperature (T_{AB}) of the phase transition between ^{3}He-B and ^{3}He-A. The vortices have a large core consisting of metastable A phase.

As a rule, vortices with a single circulation quantum ($N=1$) are stable in superfluid liquids. Vortices with larger values of N break up into elementary vortices, since their energy is proportional to $N^2 \ln(R/r_0)$ in the logarithmic approximation; here r_0 is the core radius, and R the outer cutoff parameter (the distance between vortices). The only exception to this rule so far is a nonsingular vortex with $N=2$ in ^{3}He-A (see, for example, the review in Ref. 1). This vortex competes with single-quantum vortices since it has a huge core, in which the superflow vorticity $\text{curl}\mathbf{v}_s$ has a continuous distribution. We will show that this property of the A phase—the possible existence of a continuous superflow vorticity—leads to the existence of a new type of vortex in ^{3}He-B, with a large value of N. The superflow vorticity in these vortices is distributed continuously within the core, which consists of the A phase, which is separated form the B phase outside the core by a phase boundary (the AB boundary). These vortices are locally stable if $T \ll T_{AB}$. The energy of such a vortex has three primary components (r is the core radius):

$$F = \pi r^2 (F_A - F_B) + 2\pi r \sigma_{AB} + \pi \rho_s k^2 N^2 \ln \frac{R}{r} \quad . \tag{1}$$

Here $F_A - F_B \ll |F_B|$ is the difference between the condensation energies of the A and B phases, $\sigma_{AB} \sim |F_B|\xi$ is the surface tension of the AB boundary, and the third term is the hydrodynamic energy $\frac{1}{2}\rho_s \mathbf{v}_s^2$, which collects outside the core (the hydrodynamic energy inside the core can be ignored since it is proportional to $N \ll N^2$). Here $k = \hbar/2m_3$, where m_3 is the mass of the ^{3}He atom. A minimization of the energy leads to the following equilibrium value of the core radius:

$$r_0 = \frac{\sigma_{AB}}{2(F_A - F_B)} \left[\sqrt{1 + N^2 \frac{2\rho_s k^2 (F_A - F_B)}{\sigma_{AB}^2}} - 1 \right]. \tag{2}$$

It is necessary to determine whether such a formation is stable with respect to perturbations which alter the shape of the core. An important circumstance here is that the entire superflow vorticity of the A phase is displaced toward the AB boundary because of the logarithmic repulsion of vortices of like charge inside the core. At this boundary, the vorticity of superflow forms a narrow vortex layer of size $2\pi r_0/N$. At large values of N this size is significantly greater than the coherence length ξ and thus larger than the thickness of the AB boundary. The vortex layer therefore cannot pene-

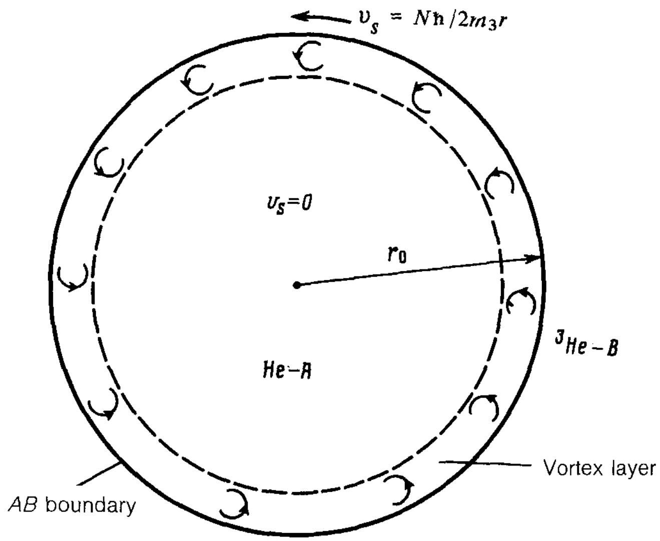

FIG. 1. Structure of a vortex with $N \gg 1$ quanta of the circulation of the superfluid velocity in ^{3}He-B near the temperature of the first-order transition between ^{3}He-B and ^{3}He-A.

trate the phase boundary. The situation is reminiscent of a charged conductor whose charge is concentrated at its surface and cannot escape the conductor. There is also a formal analogy with the electrostatics of a charged conductor. If we introduce a fictitious electric field $\mathbf{E} = \hat{z} \times \mathbf{v}_s$, where z is the direction along the vortex axis, we have three equations for this field outside the core. These equations are the same as the equations of electrostatics:

$$\vec{\nabla} \times \mathbf{E} = \nu, \qquad \vec{\nabla} \times \mathbf{E} = 0, \qquad \nu \mathbf{E} = \mathbf{F}^{ext}, \tag{3}$$

where $\nu = (\text{curl}\mathbf{v}_s)_z$ is the superflow vorticity concentrated at the core boundary. It plays the role of a surface charge density. The second equation follows from the continuity equation for the superfluid velocity $\mathbf{v}_s$, and the third equation describes the Magnus force which $\mathbf{v}_s$ exerts on the superflow vorticity (the left side). This force is canceled by the external force $\mathbf{F}^{ext}$ exerted by the boundary, i.e., by surface tension. It follows from the latter equation that the equilibrium distribution of the superflow vorticity at the surface requires as a boundary condition that there be no tangential component of the electric field at the interface. For an elliptical core, $x^2/k^2 + k^2y^2 = r^2$, the electrostatic problem of the charge distribution on a conductor has an exact solution for arbitrary k. It leads to the following change in the core energy given by (1):

$$F = \pi r^2 (F_A - F_B) + 2\pi r \sigma_{AB} k E'\left(\frac{1}{k^2}\right) + \pi \rho_s k^2 N^2 \left(\ln\frac{R}{r} + \ln\frac{2k}{k^2+1}\right), \tag{4}$$

where E' is the elliptic integral, and we have selected $k>1$. If the elliptical perturbation is small, $k-1\ll 1$, the correction to the energy is

$$\delta F=\frac{\pi}{2}(k-1)^2(3r_0\sigma_{AB}-\rho_s k^2N^2). \qquad (5)$$

A multiquantum vortex is stable only if $3r_0(N)\sigma_{AB}>\rho_s k^2N^2$. Substituting $r_0(N)$ from (2), we thus find the maximum value of N at which a vortex is still stable with respect to an elliptical perturbation:

$$N^c=\sqrt{\frac{3\sigma_{AB}^2}{2\rho_s(\frac{\hbar}{2m_3})^2(F_A-F_B)}}. \qquad (6)$$

We can now show that incorporating the following harmonics of the perturbation of the core shape does not alter expression (6). With $n=1$, this is a displacement of the core as a whole, without a change in the vortex energy. At other values $n>1$, the electrostatic problem for a small a_n can be solved by transforming to the complex variable $z=x+iy$ and then carrying out the conformal transformation $z\to z-a_n r_0^n/z^{n-1}$. Within terms quadratic in a_n, this conformal transformation restores the circular shape of the core. For the correction to the energy we find the expression

$$\delta F_n=\frac{\pi(n-1)}{2}[(n+1)r_0\sigma_{AB}-\rho_s k^2N^2]a_n^2. \qquad (7)$$

From the condition that this quantity be positive we find the maximum value of N at which the vortex is still stable with respect to the nth harmonic of the perturbation:

$$N_n^c=\sqrt{\frac{(n^2-1)\sigma_{AB}^2}{2\rho_s(\frac{\hbar}{2m_3})^2(F_A-F_B)}}. \qquad (8)$$

Since N_n^c increases with increasing n, the perturbation with the smallest value of n, i.e., the elliptical perturbation with $n=2$, is the most critical. Consequently, the maximum value of N at which a vortex is locally stable with respect to an arbitrarily small perturbation is given by (6). In the region in which the Ginzburg-Landau functional is applicable near T_c, where $\rho_s(\hbar/2m_3\xi)^2=20/3|F_B|$, we have the following expression for N^c:

$$N^c=\frac{\sigma_{AB}}{|F_B|\xi}\sqrt{\frac{9}{40}\frac{|F_B|}{F_A-F_B}}, \qquad (9)$$

According to the estimates of Refs. 2 and 3, the surface tension is $\sigma_{AB}\approx 0.85|F_B|\xi$ in this region. Consequently, as we move closer to the temperature of the first-order transition, T_{AB} the maximum charge N of a locally stable vortex increases.

Even if we are far from the transition, the difference $F_A - F_B$ is small. Furthermore, traces of this vortex core structure can be seen even in a one-quantum vortex. The core of an axially symmetric vortex with $N=1$, which exists at fairly high pressures, i.e., not far from the A-phase region, is known[1] to consist predominantly of the A phase. At lower A-phase pressures, the core becomes unstable, with the result that a phase transition occurs inside the core.

The multiquantum vortex is one example of a situation in which a metastable A phase can exist in a macroscopic volume near a stable B phase. This situation is opposite the case of nucleation, in which the stable phase is near a metastable phase. Other possible cases in which macroscopic regions of a metastable A phase exist (we intend to study such cases in the future) should arise at the boundary of a vessel.

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