Self-consistent renormalization of Chern-Simons term and correlations in system of anyons

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In the self-consistent approximation, the effect of the medium reduces to one of reducing the Chern-Simons coefficient by a factor of two, rather than erasing it altogether. Various properties of an anyon liquid, in particular, two-stage transmutation of the statistics, are discussed.

1. A new mechanism for a superconductivity in a system of anyons—two-dimensional particles with fractional statistics[3]—was recently proposed.[1,2] This new mechanism involves the appearance of a massless pole in a current–current correlation function. Using the field-theory realization of anyons as ordinary fermions which are interacting with a gauge field with a Chern-Simons action, Banks and Lukken[4] showed that a massless pole arises if and only if the action induced by the medium cancels out the seed action completely. The massless pole corresponds to a "surviving" massless Chern-Simons photon, which acquires a mass when the coefficient of the Chern-Simons action is nonzero.[5] It was shown in Refs. 6 and 7 that at a nonzero temperature T a complete cancellation does not occur, but at $T=0$ the renormalized Chern-Simons action vanishes in all orders of perturbation theory in the coupling constant. In the present letter we would like to show that actually a complete cancellation does not occur and that if the renormalized action is calculated correctly in the self-consistent approximation, then a universal finite coefficient, half the seed value, arises in the self-consistent approximation [see Eq. (10) below].

2. The system of anyons with a nonzero density is described at $T=0$ by a path integral over the fermion and gauge fields:

$$Z = \int D\psi D\bar{\psi} DA_\mu \exp(iS)$$

with this action

$$S = \int d^3x \, [\bar{\psi}(i\not{D} - m)\psi - \mu\bar{\psi}\gamma_0\psi + \frac{k}{16\pi}\epsilon_{\mu\nu\lambda}F_{\mu\nu}A_\lambda - \frac{1}{4\gamma}F^2_{\mu\nu}], \tag{1}$$

where μ is the chemical potential determined by the density of anyons, and ρ and m are the masses of a fermion and an anyon. In (1) we have introduced a term $\sim F^2_{\mu\nu}$, which arises because of quantum corrections in a medium. The anyons proper, i.e., free particles with fractional statistics, correspond to the limit $\gamma\to\infty$, but because of the finite density we have $\gamma_R^{-1}\neq 0$ (even in the nonrelativistic approximation). From classical action (1) follow the equations of motion

$$\frac{1}{\gamma}\partial_\mu F_{\mu\nu} + \frac{k}{8\pi}\epsilon_{\nu\lambda\rho}F_{\lambda\rho} = \bar{\psi}\gamma_\mu\psi \tag{2}$$

whose zeroth component is a constraint on the quantum states:

$$\frac{1}{\gamma}\partial_i\hat{F}_{i0}(x)+\frac{k}{8\pi}\epsilon_{ij}\hat{F}_{ij}(x)=\psi^+(x)\psi(x). \tag{3}$$

Taking an average of the operator equation over the space, we easily find a relation between the expectation value of the density $\hat{\rho}=(1/S)\int d^2x\psi^+(x)\psi(x)$, and that of the magnetic field, $\hat{B}=(1/2S)\int d^2x\epsilon_{ij}\hat{F}_{ij}(x)$ (or, after multiplication by S, a relation between the operator representing the total number of particles, $\hat{N}$, and the total flux operator $\hat{\Phi}$):

$$\hat{B}=\frac{4\pi}{k}\hat{\rho}. \tag{4}$$

In the mean-field approximation, which was used in Refs. 1, 2, 6, and 7, it is necessary to average (4) over the state of the system. In other words, it is necessary to examine the relationship between the renormalized matrix elements. The coefficient k must be replaced by an effective (renormalized) coefficient k_R. This renormalization follows in a nontrivial way from the form of Eq. (2), whose right side has a conserved and thus renormalization-invariant current, while the quantity $F_{\mu\nu}$ on its left side is renormalized. This renormalization is determined by the photon polarization operator, i.e., by the renormalization of the propagator. Actually, Eq. (2) and thus Eq. (4) follow from the relationship between the field and the current $A_\mu(p)=G_{\mu\nu}(p)J_\nu(p)$, where the propagator (after a renormalization) is

$$G_{\mu\nu}(p)=\frac{\gamma_R g^{\perp}_{\mu\nu}}{p^2-M^2}+i\frac{M\gamma_R\epsilon_{\mu\nu\lambda}p_\lambda}{p^2(p^2-M^2)}+\xi\frac{p_\mu p_\nu}{p^4} \tag{5}$$

and $M=\gamma_R k_R/4\pi$ is the renormalized photon mass. The renormalizations of γ and k are determined by the symmetric and antisymmetric parts of the polarization operator

$$\Pi_{\mu\nu}(p)=(p^2g_{\mu\nu}-p_\mu p_\nu)\Pi_{even}(p^2)+i\epsilon_{\mu\nu\lambda}p_\lambda\Pi_{odd}(p^2)$$

as follows:

$$k_R=k+\Pi_{odd}(0), \tag{6}$$

$$\frac{1}{\gamma_R}=\frac{1}{\gamma}+\Pi_{even}(0).$$

By virtue of the Coleman-Hill theorem,[8] $\pi_{\rm odd}(0)$ is determined exclusively by single-loop diagrams (this theorem was generalized to the case of finite densities in Ref. 6). From (6) we find a relationship between the expectation values over the state, $B=\langle\hat{B}\rangle$ and $\rho=\langle\hat{\rho}\rangle$:

$$B=\frac{4\pi}{k+\Pi_{odd}(0)}\rho. \tag{7}$$

On the other hand, the local density $\rho(x)=\langle\bar{\psi}(x)\gamma_0\psi(x)\rangle={\rm Tr}\gamma_0G(x,x)$ is itself determined by the external magnetic field B because of the explicit dependence of the fermion Green's function $G(x,y)=\Sigma_k\psi_k^*(x)\psi_k(y)/E_k$, on this field, where E_k and

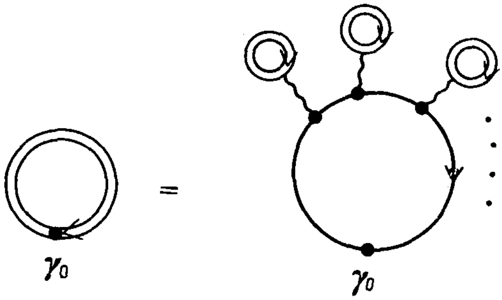

FIG. 1.

$\psi_k(x)$ are the energy eigenvalues and the wave functions of the stationary states in the external magnetic field B (Landau levels). Figure 1 shows a diagram for the density, where the tadpoles correspond to vertices representing an interaction with the external field, which depends in a self-consistent way on the density in accordance with (7). Taking an average over the space, and varying with respect to the density, we easily find the identity (Fig. 2)

$$\delta\rho = -\frac{\Pi_{odd}(0)}{k + \Pi_{odd}(0)}\delta\rho \,, \tag{8}$$

which corresponds to the identity $\delta\rho/\delta B = -1/4\times\Pi_{\rm odd}(0)$, found in Ref. 6. It follows that

$$k_R = k + \Pi_{odd}(0) = -\Pi_{odd}(0)\,. \tag{9}$$

It is easy to understand (9) on the basis of the following considerations: It follows from (7) that we have $2\pi\rho/B = k_R/2$ filled Landau levels, each of which makes a contribution of $-1/2\pi$ to the Hall conductivity $\sigma_{xy} = \Pi_{\rm odd}(0)/4\pi$ (Refs. 9 and 10). In other words, we have $\sigma_{xy} = -k_R/4\pi$. We thus find (9). In this manner we find the basic result of this paper:

$$k_R = \frac{1}{2}k \,. \tag{10}$$

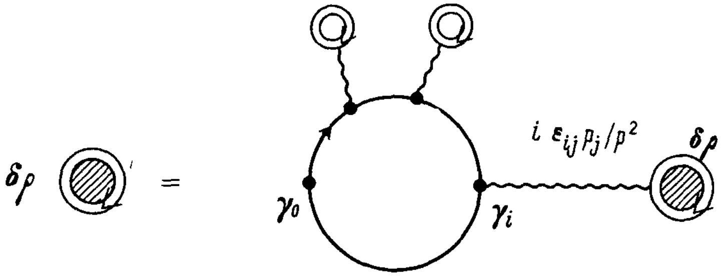

FIG. 2.

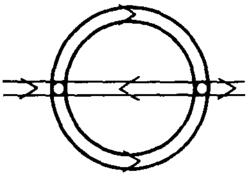

FIG. 3.

3. It has been assumed in this derivation that there is a nondegenerate ground state. This assumption is clearly valid in the case of completely filled Landau levels, which holds at $k=4N$. At other values of k, a degenerate situation with a partially filled upper level formally arises. The degeneracy is lifted, as in the case of the fractional quantum Hall effect,[9,11] when four-fermion contact interactions are taken into account. In this case, they arise because of the exchange of a massive photon, which has a finite mass $M=k_R\gamma_R$ because of (6) even in the limit $\gamma\to\infty$. From the standpoint of the one-particle spectrum, this interaction broadens a Landau level without changing the total density. The scale of this broadening is determined by the imaginary part of the operator Σ, the eigenenergy of a fermion (in leading order; Fig. 3), and is given in order of magnitude by $\Gamma\sim\rho^3/kmM^4$.

The possible disappearance of the contact interaction is interesting. In this case we find degenerate states, and a Goldstone mode may arise: The anyon liquid becomes compressible! This vanishing of the contact interaction may occur when a certain relationship holds between the masses of the fermion, m, and of the photon M: $m=M$ or, more precisely, $2\mu M=1$, where μ in this case is the magnetic moment of a fermion. As was shown in Ref. 12, this point separates the region of repulsion, $2\mu M<1$, from the region of attraction, $2\mu M>1$, of two fermions. In the latter case, the system becomes unstable. It may be that the transition region of a "zero short-range interaction" gives rise to massless excitations. This matter definitely requires further and careful study. Also of interest are cases in which a degeneracy arises among a finite number of states; in such cases, a Goldstone mode does not arise, but there may be corrections $o(k)$ to law (10).

4. We conclude with a discussion of some consequences of renormalization (10). Let us examine the interchange of two fermions at a distance r. The phase factor which arises here is $\exp(\pi i-2\pi i c(r))$, where

$$c(r)=\frac{1}{k(r)}(1-M(r)rk_1(rM(r))), \tag{11}$$

and $M(r)$ and $k(r)$ are the values of the mass and the coefficient k at the characteristic scale r. Here $k(0)=k$ and $k(\infty)=k/2$, and the transition occurs at scales of order $\rho^{-1/2}$. The characteristic scale over which the second term in parentheses in (11) becomes small is M^{-1}; this term is always smaller than the first. As a result, there is a two-stage transmutation of the statistics. Specifically, c changes from 0 to $1/k$ over a scale M^{-1} and then from $1/k$ to $2/k$ in the infrared region. In the case of "semions,"[13] for which we have $k=4$, the phase factor is $\exp(\pi i-2\pi i\cdot\frac{2}{4})=1$, and the statistics are

Bose statistics. If a BCS condensate forms in this case, the effective theory will be P-even, since the value $c = 1/2$ corresponds to exactly the points at which P parity is restored. A second transmutation of this sort does not occur in the case of the doubly charged anyon liquid discussed in Ref. 14, since in this case there is no average field, and there is simply no renormalization as in (10). We note in conclusion that the pressence of this renormalization in a singly charged liquid leads to the appearance of two scales, M^{-1} and $\rho^{-1/2}$, and to the interesting interpretation of the effective action in the infrared region as a topologically massive theory with spontaneous symmetry breaking (the Higgs mechanism). In this case the anyons are described by vortex solutions in this phase (an anyon as a soliton). This picture will be drawn in more detail in a separate publication by the present authors.

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