Superfluidity in system with fermion condensate

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The properties of Fermi systems beyond the phase transition point, at which the group velocity of the quasiparticles changes sign on the Fermi surface, are analyzed. A Fermi condensate arises in the new phase: The energies $\epsilon(\mathbf{p})$ of quasiparticles with momenta $p_{1c} < p < p_{2c}$ ($p_{1c} < p_F, p_{2c} > p_F$) turn out to be identical and equal to the chemical potential μ. If a Cooper pairing can occur in this phase, the gap Δ is a linear function of the pairing constant λ.

The theory of a Fermi liquid[1] is based on a representation of the system as a gas of interacting quasiparticles, the number of which is, according to the Landau-Lüttinger theorem, equal to the number of particles. In this theory, the energy of the system, E, is a functional of the quasiparticle distribution function $n(\mathbf{p})$, and for any variation in $n(\mathbf{p})$ which conserves the number of particles we have

$$\delta E = \int (\epsilon(\mathbf{p}, n(\mathbf{p})) - \mu)\delta n(\mathbf{p}) \frac{d^3p}{(2\pi)^3} V . \qquad (1)$$

Here $\epsilon[\mathbf{p}, n(p)]$ is the energy of an individual quasiparticle, and μ is the chemical potential of the system.

Since the classification levels of the liquid and the gas coincide, the distribution $n(\mathbf{p})$ for the ground state of the system at a temperature $T = 0$ is a Fermi "step" distribution $n = n_F(\mathbf{p}) = \theta(p - p_F)$ (p_F is the Fermi momentum, and $\rho = p_F^3/3\pi^2$), is the density); here $\mu = \epsilon(p_F, n_F)$.

A Landau Fermi liquid has three primary distinguishing features: (1) a discontinuity at the point $p = p_F$ in the momentum distribution of the particles of the medium;[2] (2) a linear temperature dependence of the heat capacity, $C(T)$, in the limit[3] $T \to 0$; (3) a logarithmic behavior of the amplitude for the scattering of quasiparticles with a resultant momentum of 0 near the Fermi surface.[4] As a result, the gap in the spectrum of one-particle excitations excited by the Cooper pairing of the particles of the medium is exponentially small in a system with a weak attraction.

It is simple to show that a necessary condition for the validity of the Fermi-liquid theory[1] is that the group velocity (v_g) of the quasiparticles calculated for a filling $n_F(\mathbf{p})$ be nonnegative on the Fermi surface: $v_g(p_F) = (d\epsilon/dp)_{p=p_F}$. By virtue of the Pauli principle, only those variations of $n_F(\mathbf{p})$ whose sign is the same as that of the difference $(p - p_F)$ are allowed in (1). As long as the condition $v_g(p_F) > 0$ holds, the signs of $(p - p_F)$ and $[\epsilon(\mathbf{p}) - \mu]$ will be the same, since we have $\mu = \epsilon(p_F)$. The integrand in (10) is then positive for all variations of $n_F(\mathbf{p})$. When the condition $v_g(p_F) < 0$, holds, in contrast, there are variations $\delta n_F(\mathbf{p})$ for which the relation $\delta E < 0$ holds; in this case, the Fermi filling will inevitably undergo a restructuring.

This transition has nothing in common with the transition which results from a violation of the Pomeranchuk stability condition for the effective mass[4] M^*, since the latter occurs as $M^* \to 0$; i.e., the fermions "get lighter" near the transition point. The transition under consideration here, in contrast, occurs as $M^* \to \infty$; i.e., the fermions "get heavier."

What happens to a system of "heavy" fermions beyond the phase transition? The quasiparticle distribution function $n(\mathbf{p})$ now acquires some freedom. The minimum of the energy is thus reached with a new filling $n_0(\mathbf{p})$, which is determined in a certain interval of momenta $p_{1c} < p < p_{2c}$ $[p_{1c} \langle p_F;\ p_{2c} \rangle\ p_F]$ by the equation

$$\frac{\delta E_0(n(\mathbf{p}))}{\delta n(\mathbf{p})} \equiv \mu \,. \tag{2}$$

Outside this interval, $n_0(\mathbf{p})$ is the same as $n_F(\mathbf{p})$.

Since the energy of a quasiparticle, $\epsilon_q[\mathbf{p}, n_0(\mathbf{p})] = E_0(N+1, \mathbf{p}) - E_0(N)$ is again the same as $\epsilon[\mathbf{p}, n_0(\mathbf{p})] = \delta E_0/\delta n(\mathbf{p})$, in this case, some of the quasiparticles in the new ground state will again have the same energy $\epsilon(\mathbf{p}) = \mu(p_{1c} < p < p_{2c})$. By analogy with a Bose liquid, such systems might be called "fermion-condensate systems." The problem of finding a microscopic description of these systems will have to be studied separately. Analysis shows that the properties of fermion-condensate systems are sharply different from the ordinary properties. That this is true can also be seen in simple models with effective phenomenological functionals $E_0(n(\mathbf{p}))$, e.g.,

$$E_0(n(\mathbf{p})) = 2\int \frac{p^2}{2M} n(\mathbf{p}) \frac{d^3p}{(2\pi)^3} + g \iint n(\mathbf{p}) n(\mathbf{p}') \, U(\mathbf{p}-\mathbf{p}') \frac{d^3p\, d^3p'}{(2\pi)^6} \tag{3}$$

with the standarad normalization of $n_0(\mathbf{p})$ to the density ρ. If the momentum $\mathbf{p}$ is interpreted as a coordinate in ordinary space, and $n_e = 2n_0(\mathbf{p})/(2\pi)^3$ as the particle density at this coordinate, the problem reduces to one of finding the optimum distribution of a given number of particles which are interacting by means of a pair potential U in an elastic external field under the further requirement that the density n_e nowhere exceed the critical density $n_c = 1/4\pi^3$. If $U(q) = g/q$, the interaction will be a "Coulomb" interaction, and the solution will be particularly simple. The sole dimensionless parameter of the problem is $\xi = gM/6\pi^2$. As long as it is small, the "elastic forces" which realize the Fermi step $n_F(\mathbf{p})$ will dominate. The transition point corresponds to the value $\xi_c = 1$, at which the effective mass M^* becomes infinite. With $\xi > 1$, $p_{1c} = 0$, $p_{2c} = p_0$,

$$n_0(p) = \theta(p - p_0)/\xi \qquad (p_0 = p_F \xi^{1/3}) \,, \tag{4}$$

and a quasiparticle energy

$$\epsilon(p) = \begin{cases} \mu & p < p_0 \\ (p^2/2 + p_0^3/p)/M & p > p_0 \end{cases} \tag{5}$$

we find

$$\mu = \frac{3p_F^2}{M} \xi^{2/3}.$$

In this model, (1) all the particles are simultaneously in the condensate just beyond the transition point $\xi_c = 1$, (2) the discontinuity remains in the filling $n_0(\mathbf{p})$, but shifts to the point $p_0 = p_F \xi^{1,3} > p_F$, and (3) the group velocity $v_g(p_0) = (d\epsilon/dp)_{p=p_0}$ vanishes at this point (this is a simple consequence of the equilibrium between the "elastic" and "Coulomb" forces). Consequently, near $p = p_0$ the quasiparticle energy is $\epsilon(p) \sim (p - p_0)^2$, and the heat capacity has the behavior $C(t) \sim T^{1/2}$ as $T = 0$. Analytic results can also be found for a system with a "Yukawa" potential $U(q) = ge^{-\beta q}/q$. Here, in contrast with the preceding case, the Fermi condensate which arises beyond the transition point (where, as before, $M^* = \infty$) initially fills a narrow zone near the old Fermi surface, gradually expanding with increasing coupling constant g. The quasiparticle distribution $n_0(\mathbf{p})$ in this zone is parabolic: For $p_{1c} < p < p_{2c}$, where $\epsilon(\mathbf{p}) = \mu$, we have

$$n_0(\mathbf{p}) = n_{01} + (n_{02} - n_{01})(p^2 - p_{2c}^2)/(p_{2c}^2 - p_{1c}^2), \tag{6}$$

and $d\epsilon(\mathbf{p})/dp$ is a continuous function of p for all p. Beyond the transition point, the distribution $n_0(\mathbf{p})$ thus has two discontinuities, rather than a single one, and the heat capacity has the behavior $C(T) \sim T^{1/2}$ as $T \to 0$.

Examining the conditions for stability of the resulting ground state, we can see that any of them will be violated because of a multiple degeneracy of the ground state. We restrict the present discussion to the case in which a singlet pairing instability due to attraction in the particle–particle channel is dominant and leads to the formation of Cooper pairs from condensate particles. The gap Δ in the spectrum of singlet-particle excitations is found by means of a Bogolyubov transformation. The condensate particles dominate this gap. The result turns out to be the same as for a system of particles in a common, degenerate, isolated j level:[5]

$$\Delta = \frac{\lambda}{2} \int_{p_{1c}}^{p_{2c}} \sqrt{n_0(\mathbf{p})(1 - n_0(\mathbf{p}))} \frac{d^3p}{(2\pi)^3}, \tag{7}$$

where $n_0(\mathbf{p})$ is given by (6). We see that the relationship between Δ and the pairing constant λ is linear, and the value of Δ is not exponentially small in these systems.

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