Attraction of gluons in (2+1)-dimensional topologically massive gauge theory

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The attraction amplitude of gluons of like charge in a topologically massive SU(2) gauge theory is calculated. The sign of this amplitude corresponds to an attraction on the mass shell.

Topologically massive gauge theories[1-3] are presently the subject of active research. It was shown in Ref. 4 that an attraction of two electrons with a mass m smaller than the topological mass of a photon, M, arises in a topologically massive electrodynamics.

In the present letter we show that an attraction of particles of like charge also occurs in the non-Abelian theory which was first examined in Ref. 3. For simplicity we consider the case of the SU(2) theory; the generalization to the case of an arbitrary Lie group G is obvious.

We consider a $(2+1)$-dimensional SU(2) gauge theory with the Lagrangian

$$L = -\frac{1}{2\gamma}\,\mathrm{Tr}F^2_{\mu\nu} + \frac{k}{4\pi}\epsilon_{\mu\nu\lambda}\,\mathrm{Tr}\left(A_\mu\partial_\nu A_\lambda + \frac{2}{3}A_\mu A_\nu A_\lambda\right), \qquad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$; $A_\mu = A^a_\mu T^a$ has the dimensionality of a mass; and $k \in Z$ is a dimensionless integer, as follows from the condition of gauge invariance of the action $S = \int d^3x L$ under "large" gauge transformations. The theory is defined in a space–time with a signature $(+, -\ -)$; the generators of the gauge group are normalized by the condition $\mathrm{Tr}T^aT^b = (1/2)\delta^{ab}$.

The part of Lagrangian (1) which is quadratic in the fields is the sum of U(1)-invariant Lagrangians:

$$L = -\frac{1}{4\gamma}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{k}{8\pi}\epsilon_{\mu\nu\lambda}A^a_\mu\partial_\nu A^a_\lambda, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu . \qquad (2)$$

The equations of motion

$$\partial_\mu F^a_{\mu\lambda} + \frac{M}{2}\epsilon_{\mu\nu\lambda} F^a_{\mu\nu} = 0, \qquad M = \frac{\gamma k}{4\pi} \tag{3}$$

lead to plane-wave solutions

$$\begin{gathered} A^a_\mu(x) = \sum_{\mathbf{p}} a^a_{\mathbf{p}} \sqrt{\frac{M}{2\epsilon_{\mathbf{p}} V}}\, e_\mu \exp\,(-ip_\mu x^\mu) + \text{c.c.}, \\ p_0 = \epsilon_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + M^2}, \end{gathered} \tag{4}$$

where e_μ, the polarization vector, is given by[5]

$$e_\mu = \begin{pmatrix} e_0 \\ e_1 \\ e_2 \end{pmatrix} = \frac{1}{\sqrt{2}M\,|\mathbf{p}|} \begin{pmatrix} \mathbf{p}^2 \\ \epsilon_{\mathbf{p}} p_1 + iMp_2 \\ \epsilon_{\mathbf{p}} p_2 - iMp_1 \end{pmatrix}; \qquad \bar{e}_\mu e_\mu = -1\,. \tag{5}$$

The field A_μ is transverse, has one degree of freedom, and is normalized to a single particle of mass M in a volume V.

The small parameter in the perturbation theory is the quantity $\alpha = 4\pi/|k|$, $|k| \gg 1$. It is thus convenient to redefine the fields: $A_\mu \to \sqrt{\alpha} A_\mu$. In this normalization, Feynman's rules take the following form: The propagator for a free field in this theory, in the transverse gauge, is

$$\frac{i}{M}\Pi_{\mu\nu}(p) = [g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} + \frac{iM}{p^2}\epsilon_{\mu\nu\lambda} p_\lambda]\frac{1}{p^2 - M^2}\,.$$

This three-gluon vertex differs from the ordinary momentum-independent term:

$$M\Gamma^{a\,bc}_{\mu\nu\lambda}(p,q,r) = f^{abc}[iM\epsilon_{\mu\nu\lambda} + g_{\mu\nu}(p-q)_\lambda + g_{\nu\lambda}(q-r)_\mu + g_{\lambda\mu}(r-p)_\nu\,],$$

where p, q, r are momenta emerging from a vertex. The four-gluon vertex in this theory is the same as the usual one.

We replace A^1_μ and A^2_μ by fields which are charged with respect to $A^3_\mu = Z_\mu$:

$$W^\pm_\mu = A^1_\mu \pm iA^2_\mu\,.$$

The scattering amplitude $M(W^+W^+ \to W^+W^+)$ is determined by three Feynman diagrams (Fig. 1). Omitting the lengthy exact expression for the amplitude M, we write the asymptotic value in the c.m. frame $\mathbf{p}_1 = \mathbf{p}$, $\mathbf{p}_2 = -\mathbf{p}$ in the limit $\mathbf{q} \to 0$, $\mathbf{p} \to 0$, where $\mathbf{q}$ is the momentum transfer, $\mathbf{q}^2 = -2\mathbf{pq}$:

$$M^{++} \approx \frac{2\alpha}{M}\left(3 + i\,\frac{[\mathbf{p},\mathbf{q}\,]}{\mathbf{q}^2}\right) + o(\mathbf{p},\mathbf{q}). \tag{6}$$

The sign of ReM^{++} corresponds to an attraction of gluons of like charge. It can be

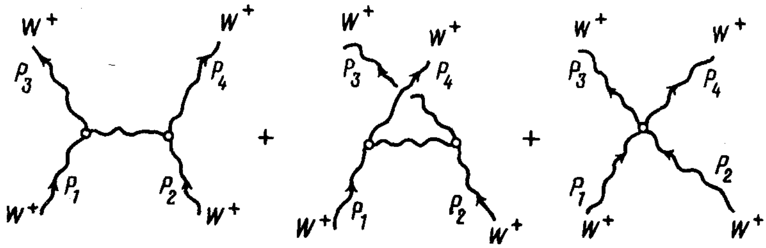

FIG. 1.

shown that this attraction persists at nonzero, finite $\mathbf{p}$. By virtue of the C invariance of the theory we have $M(W^-W^- \to W^-W^-) = M(W^+W^+ \to W^+W^+)$. The amplitude for the scattering of a particle by an antiparticle in the same limit is

$$M^{+-} \approx -\frac{2\alpha}{M}\left(1 + i\frac{[\mathbf{p},\mathbf{q}]}{\mathbf{q}^2}\right) + o(\mathbf{p},\mathbf{q}), \tag{7}$$

where $\mathrm{Re}M^{+-} < 0$ (a repulsion).

The scattering amplitude $M(ZW^{\pm} \to ZW^{\pm})$ in the limit $\mathbf{q} \to 0$, $\mathbf{p} \to 0$ is

$$M^{0\pm} \approx \frac{2\alpha}{M}\left(1 + i\frac{[\mathbf{p},\mathbf{q}]}{\mathbf{q}^2}\right) + o(\mathbf{p},\mathbf{q}). \tag{8}$$

The sign of $\mathrm{Re}M^{0\pm}$ ($\mathrm{Re}M^{0\pm} > 0$) corresponds to an attraction of Z to $W^{\pm}$.

The terms singular in $\mathbf{q}$ in the imaginary parts of amplitudes (6)–(8) result from a nondispersive pole in the propagator associated with the Bohm-Aharonov interaction of charges.[2,3,5] It can be shown that in the transition to an effective Hamiltonian for W bosons this singularity drops out of a calculation of the expectation value of the energy of an N boson state. For a certain family of trial wave functions, the energy density is $\epsilon = nM - (6\alpha/M)n^2$, where $n = N/V$ is the density of W^+ (or W^-) bosons. We see that in the case $n \gg M^2/\alpha$ we would have $\epsilon < 0$; i.e., the vacuum would be unstable with respect to the production of W^+W^- pairs followed by a separation of phases. Whether this is actually the case depends on the components of the energy density ϵ which are of higher order in the density n. If the vacuum is nevertheless unstable, what new ground state arises? This question will have to go unanswered for the time being. One possibility for a stabilization of the system would run as follows: The two-gluon scattering amplitude should generally depend on the density n, and the effective constant λ might change sign at some n_c. As a result, the attraction would give way to a repulsion. In such a case, the ground state of the theory would be a state with a finite density $n \sim n_c$.

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