Twistors, harmonics, and zero-super-p-branes

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A new approach to the description of extended supersymmetric entities is developed. Twistor-like harmonic variables are used. The theory of zero-superstrings and zero-supermembranes is used as an example to propose an alternative quantization scheme, which combines the concept of harmonics and the BRST-BFV quantization method, modified on the basis of the idea of converting constraints of the second kind into constraints of the first kind.

Recent progress toward the solution of the problem of the covariant quantization of superparticles and superstrings has shown that harmonic and twistor variables can be used effectively for a covariant separation of constraints of the first and second kinds.[1–7] In a theory of superparticles with $p^2=0$, twistors $\mathbb{Z}_{\hat{\alpha}}=(\lambda_\alpha,\bar{\mu}^{\dot{\alpha}})$ and $(\overline{\mathbb{Z}_{\hat{\alpha}}})$ arise in the Cartan-Penrose representation $p_{\alpha\dot{\alpha}}=\lambda_\alpha\bar{\lambda}_{\dot{\alpha}}$ for the 4-momentum $p_{\alpha\dot{\alpha}}=\sigma^m_{\alpha\dot{\alpha}}p_m$. A corresponding representation for the zero vectors Π^m_+,Π^m_- in superstring theory[1] was proposed in Ref. 8. In the present letter we propose a covariant sheet generalization of the Cartan-Penrose representation for the momentum density $\mathcal{P}^\mu_{\alpha\dot{\alpha}}$ $[\mu=(\tau,M);\ M=1,\ldots,p]$ of an extended supersymmetric entity of dimensionality $p\geqslant 1$, a zero-super-p-brane,[9] which is moving in a space-time of dimensionality $D=4$ (the case $p=1$ corresponds to zero-superstrings[9,10]). It is written in the form

$$\mathcal{P}^\mu_{\alpha\dot{\alpha}}=\rho^{(-|+)\mu}(\tau,\vec{\sigma})v^-_\alpha(\tau,\vec{\sigma})\bar{v}^+_{\dot{\alpha}}(\tau,\vec{\sigma}). \tag{1}$$

Expression (1) makes it possible to go over to a new twistor-like formulation of the theory of zero-super-p-branes with an N-expanded SUSY defined by the action

$$S=-\frac{1}{2}\int d\tau d^p\vec{\sigma}\rho^{(-|+)\mu}(\tau,\vec{\sigma})v^-_\alpha(\tau,\vec{\sigma})\bar{v}^+_{\dot{\alpha}}(\tau,\vec{\sigma})\,\omega^{\dot{\alpha}\alpha}_\mu(\tau,\vec{\sigma}) \tag{2}$$

$$\omega^{\dot{\alpha}\alpha}_\mu=\omega^m_\mu\tilde{\sigma}^{\dot{\alpha}\alpha}_m,\quad \omega^m_\mu\equiv\partial_\mu x^m-(i\partial_\mu\theta^\alpha_i\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}i}+\text{c.c.}),\quad (i=1,\ldots,N)$$

which permits a direct covariant BRST quantization by the BFV scheme, modified in accordance with Refs. 11–13. The boson variables $v_\alpha^\mp \equiv v_\alpha^{(0|\mp 1)}$ and $\bar{v}_{\dot\alpha}^\pm \equiv \bar{v}_{\dot\alpha}^{(\pm 1|0)} = \overline{(v_\alpha^\mp)}$ which are used in (1) and (2)—the so-called Lorentz harmonics[14]—are constrained by the conditions $\Xi \equiv v^{\alpha -} v_\alpha^+ - 1 = 0, \bar{\Xi} \equiv \bar{v}_{\dot\alpha}^- \bar{v}^{+\dot\alpha} - 1 = 0$ and are related to the twistors $\mathbb{Z}_{\hat\alpha}$ and $(\overline{\mathbb{Z}_{\hat\alpha}})$ by[2)] $\lambda_\alpha = v_\alpha^- (\lambda^\beta \mu_\beta)^{1/2}, \mu_\alpha = v_\alpha^+ (\lambda^\beta \mu_\beta)^{1/2}$. Representation (1) is found from the Cartan-Penrose representation, rewritten in terms of[14] v_α and $\bar{v}_{\dot\alpha}$, i.e., $p_{\alpha\dot\alpha} = \rho^{(-|+)} v_\alpha^- \bar{v}_{\dot\alpha}^+$ by the replacement of $\rho^{(-|+)}$ in it by the sheet vector $\rho^{(-|+)\mu}$. Representation (2) uses the coordinates X^A of an expanded harmonic superspace $X^A = (x^m, \theta_i^\alpha, \bar\theta^{\dot\alpha i}, v_\alpha^\mp, \bar{v}_{\dot\alpha}^\pm, \rho^{(-|+)\mu})$. This approach does not increase the number of physical degrees of freedom, since the harmonics v, $\bar v$ are, by virtue of the equations of motion, purely gauge degrees of freedom. They do, however, play a decisive role below in the Lorentz-covariant separation of the constraints of zero-super-p-branes in (2) into irreducible constraints of the first and second kinds. The constraints of the first kind in terms of $X^A_{(\rho)}$ and the conjugate canonical momenta $\mathscr{P}_A = (\mathscr{P}_m, \pi_\alpha^i, \bar\pi_{i\dot\alpha}, \mathscr{P}_\alpha^\pm, \bar{\mathscr{P}}_{\dot\alpha}^\mp, \mathscr{P}_\mu^{(+|-)}) \equiv (-1)^{A+1}(\partial L/\partial \dot{X}^A), \{X^A(\boldsymbol{\sigma}), \mathscr{P}_B(\boldsymbol{\sigma}')\} = \delta_B^A \delta_{\sigma\sigma'}$ $[\delta_{\sigma\sigma'} \equiv \delta^p(\boldsymbol{\sigma} - \boldsymbol{\sigma}')$ is a p-dimensional δ-function$]$ are

$$D^{-i} \equiv v^{\alpha -} D^i_\alpha \approx 0, \quad \nabla^{(0)} \equiv v^{\alpha +} \mathscr{P}^-_\alpha - v^{\alpha -} \mathscr{P}^+_\alpha + \rho^\tau \mathscr{P}^{(\rho)}_\tau \approx 0,$$

$$\nabla^{-2} \equiv v^{\alpha -} \mathscr{P}^-_\alpha \approx 0, \quad \mathscr{P}^{(+|-)} \equiv v^{\alpha -} \bar{v}^{\dot\alpha +} \mathscr{P}_{\alpha\dot\alpha} \approx 0, \quad \mathscr{P}^{(\rho)}_M \approx 0,$$

$$T_M \equiv \frac{1}{2} \omega_M^{\dot\alpha\alpha} \mathscr{P}_{\alpha\dot\alpha} + [\partial_M \theta_i^\alpha D^i_\alpha + \partial_M v^{\alpha -} \mathscr{P}^+_\alpha + \partial_M v^{\alpha +} \mathscr{P}^-_\alpha + \text{c.c.}] - \rho^\tau \partial_M \mathscr{P}^{(\rho)}_\tau \approx 0 \tag{3}$$

and complex-conjugate relations $\bar{D}_i^{\,+} \approx 0, -\nabla^{(0)} \approx 0, \bar\nabla^{+2} \approx 0$. The spinor constraints D^i_α, $\bar{D}_{\dot\alpha u}$ in (3) have the standard form $D^i_\alpha = -\pi^i_\alpha + i\mathscr{P}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}_i = \overline{(D_{\dot\alpha i})}$. Constraints (3) on the Poisson brackets $\{...,...\}$ for the algebra

$$\{D^{-i}(\vec\sigma), \bar{D}_j^+(\vec\sigma')\}_+ = -2i\mathscr{P}^{+-}\delta^i_j \delta_{\vec\sigma\vec\sigma'}, \quad \{\nabla^{(0)}(\vec\sigma), Y_\Lambda(\vec\sigma')\} = q_R(Y_\Lambda) Y_\Lambda \delta_{\vec\sigma\vec\sigma'}$$

$$\{T_M(\vec\sigma), Y_\Lambda(\vec\sigma')\} = -Y_\Lambda(\vec\sigma')\partial'_M \delta_{\vec\sigma\vec\sigma'}, \quad \{T_M(\vec\sigma), T_N(\vec\sigma')\} = T_M(\vec\sigma')\partial'_N \delta_{\vec\sigma\vec\sigma'} - (\vec\sigma \leftrightarrow \vec\sigma'), \tag{4}$$

where Y_Λ denotes any of the constraints of the first kind in (3), while $q_R(Y_\Lambda)$ is the charge of this constraint with respect to $U_R(1)$ of the subgroup: $(q_R(T_M, \bar{\bar\nabla}^{+2}, \bar{D}_i^{\,+}, \hat{\bar\nabla}^{(0)}, \hat\nabla^{(0)}) = 0, q_R(D^{-i}) = q_R(\mathscr{P}^{(+|-)}) = -1, q_R(\hat\nabla^{-2}) = -2)$. All the other brackets of constraints (3) either vanish or can be found from (4) by complex conjugation $q_R(Y_\Lambda) = -q_L(\bar{Y}_{\bar\Lambda})$. The generators $\mathscr{P}^{+-}$ and T_M correspond to a reparametrized symmetry of action (2), D^{-i} and $\bar{D}_i^{\,+}$ correspond to the Siegel SUSY, $\hat\nabla^{(0)}$ and $\hat\nabla^{(0)}$ correspond to a local $SO(1,1) \otimes SO(2)$, while $\hat\nabla^{-2}$ and $\hat{\bar\nabla}^{+2}$ correspond to local shifts of the v^+ and $\bar{v}^-$ harmonics.

The constraints of the second kind, reduced to canonical sympletic form, are written as the relations

$$(D^{+i})_{diag} \equiv \frac{v^{+\alpha} D^i_\alpha}{\sqrt{(\Xi+1)}} \approx 0, \quad \chi \equiv v^{\alpha +}\mathcal{P}^-_\alpha + v^{\alpha -}\mathcal{P}^+_\alpha + \rho^\tau \mathcal{P}^{(\rho)}_\tau \approx 0,$$

$$\nabla^{+2} \equiv v^{\alpha +}\mathcal{P}^+_\alpha \approx 0, \quad (\mathcal{P}^{--})_{diag} \equiv \frac{\mathcal{P}^{--}}{\sqrt{(1+\Xi)(1+\bar{\Xi})}}, \quad \Xi \equiv v^{\alpha -} v^+_\alpha - 1 \approx 0, \tag{5}$$

$$(\rho^\tau)_{diag} \equiv \frac{\rho^\tau - \mathcal{P}^{-+}}{\sqrt{(1+\Xi)(1+\bar{\Xi})}}, \quad (\mathcal{P}^{(\rho)}_\tau)_{diag} \equiv \sqrt{(1+\Xi)(1+\bar{\Xi})}\,\mathcal{P}^{(\rho)}_\tau$$

and the complex conjugates. Here $\mathscr{P}^{-+} \equiv v^{\alpha +} \bar{v}^{\dot\alpha -} \mathscr{P}_{\alpha\dot\alpha}$, $\mathscr{P}^{--} \equiv -v^{\alpha -} \bar{v}^{\dot\alpha +} \mathscr{P}_{\alpha\dot\alpha} = \overline{(\mathscr{P}^{++})}$. The symplectic structure of the "diagonalized" constraints of the second kind in (5) is described by the following nonvanishing Poisson brackets $\{J^{(-|+)} \equiv [\mathscr{P}^{(-|+)}/\sqrt{(1+\Xi)(1+\bar{\Xi})}]\}$:

$$\{(D^{+i})_{diag}(\vec{\sigma}), (\bar{D}^-_j)_{diag}(\vec{\sigma}')\} = -2iJ^{-+}\delta^i_j \delta_{\vec{\sigma}\vec{\sigma}'}, \quad \{(\mathcal{P}^{(\rho)}_\tau)_{diag}(\vec{\sigma}), (\rho^\tau)_{diag}(\vec{\sigma}')\} = \delta_{\vec{\sigma}\vec{\sigma}'},$$

$$\{\nabla^{+2}(\vec{\sigma}), (\mathcal{P}^{--})_{diag}(\vec{\sigma}')\} = -J^{-+}\delta_{\vec{\sigma}\vec{\sigma}'}, \quad \{\Xi(\vec{\sigma}), \chi(\vec{\sigma}')\} = -2(1+\Xi)\delta_{\vec{\sigma}\vec{\sigma}'} \tag{6}$$

and complex conjugates. Here $\{J^{(-|+)}(\sigma)$ [constraints of the second kind in (5)]$\} = 0$.

A central problem in the BRST-BFV quantization scheme[11–13] is that of Abelizing the algebra of constraints of the second kind by adding new pairs of canonical variables. In the case at hand, the constraints of the second kind in (5) are already in sympletic form, so their conversion into Abelized constraints poses no difficulty. It leads to some new constraints of the first kind:

$$(D^{+i})_{AB} = (D^{+i})_{diag} + J^{1/2}\psi^i \approx 0, \quad \chi_{AB} \equiv \chi - p_2 \approx 0, \quad \nabla^{+2}_{AB} \equiv \nabla^{+2} + p_1 \approx 0,$$

$$(\mathcal{P}^{--})_{AB} \equiv (\mathcal{P}^{--})_{diag} - Jq_1 \approx 0, \quad \Xi_{AB} \equiv e^{2q_2}(1+\Xi) - 1 \approx 0,$$

$$(\mathcal{P}^{(\rho)})_{AB} \equiv (\mathcal{P}^{(\rho)}_\tau)_{diag} + p_3 \approx 0, \quad (\rho^\tau)_{AB} \equiv (\rho^\tau)_{diag} - q_3 \approx 0 \tag{7}$$

and complex conjugates. The nonvanishing Poisson brackets of the new canonical variables, $(\psi^i, \bar{\psi}_i)$, (q^r, p_r), $q^r \equiv (q_1, q_2, q_3, \bar{q}_1, \bar{q}_2, \bar{q}_3)$, $q_1 \equiv q^{(0|-2)}$, $q_2 \equiv q_\chi$, $q_3 \equiv (q^{(-|+)}$ $\bar{q}_1 \equiv \bar{q}^{(+2|0)} \ldots)$ are the form $\{\psi^i(\sigma), \bar{\psi}_j(\sigma')\}_+ = 2\delta^i_j \delta_{\sigma\sigma'}$, $\{q^r(\sigma), p_r(\sigma')\} = -\delta_{\sigma\sigma'}$. The next important step is to modify the constraints of the first kind by means of the new variables (q^r, p_r), $(\psi^i, \bar{\psi}_i)$ in such a way that their Poisson brackets with Abelized constraints (7) vanish. We do not have room here to write out the modified constraints in (3) which have this property. After going through this procedure, we obtain an expanded algebra of constraints of the first kind with structure functions which are nonzero for only the modified constraints in (3). These structure functions are expressed in terms of the constraints in (5) and the factor $J^{(+|-)}$. This circumstance erases all the restrictions on the construction of a BRST charge and on a quantization of the theory of zero-super-p-branes described by action (2) by means of a modified BFV scheme.[12]

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[2)] The harmonics $v_\alpha^{\mp}, \bar{v}_{\dot\alpha}^{\pm}$ parametrize the factor-space $SL(2\mathbb{C})/[U(1)]^c$ (Ref. 14), where $[U(1)]^c \approx U_L(1) \otimes U_R(1) \approx \otimes SO(1,1) \otimes SO(2)$ is a complexification of the $U(1)$ group. The "indices" $(0|\mp 1)$ and $(\pm 1|0)$ of the harmonics $v^{\mp}$ and $\bar{v}^{\pm}$ specify the charges of these harmonics with respect to the local groups $U_R(1)$ $q_R(v^{\mp}) = -q_L(\bar{v}^{\pm}) = \mp 1; q_R(\bar{v}) = q_L(v) = 0$, which are determined by multiplying a matrix of subgroup $[U(1)]^c$ with $SL(2\mathbb{C})$ by the $SL(2\mathbb{C})$ matrix $V_\alpha^{(\beta)} = (v^{(0|+1)}, v_\alpha^{(0|-1)})$. For brevity, $U_L(1)$ and $U_R(1)$, the charges of certain variables, e.g., $\rho^\tau \equiv \rho^{(-|+)\tau}$ are omitted below [see (2), etc.].

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