Electron pairing due to Friedel oscillations in two-species system

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A two-component electron gas consisting of light and heavy electrons of different densities is analyzed. Friedel oscillations of the Coulomb interaction of the heavy electrons can lead to a pairing in coordinate space or momentum space.

The Coulomb interaction in a degenerate electron gas is an oscillatory interaction, as is well known. The presence of attractive regions in the potential can result in a pairing of electrons with angular momenta $l \neq 0$ (Refs. 1 and 2). At short range, repulsion interferes with $l = 0$ pairing. In a system with two types of electrons, with masses m and M, however, the presence of the large parameter M/m would allow heavy electrons to pair in any state, including the s state.[3]

Two scenarios have been proposed for superconductivity. In the standard Cooper scenario, a Bose condensation of pairs occurs at the same time that the pairs appear. In the scenario of Ref. 4, bielectrons initially form and subsequently undergo a Bose condensation. Unfortunately, the attraction of electrons generally results in their crystallization, which prevents superconductivity. We will show in this paper that if there is no conversion of heavy electrons into light electrons, then both the formation of heavy bielectrons and Cooper pairing are possible.

We consider the interaction of a small number of heavy electrons with each other against the background of delocalized light electrons. This interaction is described by the potential

$$V(r) = \int \frac{d^3q}{(2\pi)^3} \frac{4\pi e\, e^{i\mathbf{qr}}}{\epsilon\,[q^2 + \kappa^2 (\frac{1}{2} + \frac{4p_F^2 - q^2}{2p_F q} \ln | \frac{2p_F + q}{2p_F - q} |)]} , \qquad (1)$$

$$\kappa^2 = \frac{4}{\pi} m e^2 p_F .$$

In the limit $r \gg \hbar/p_F$, expression (1) has the asymptotic behavior

$$V(r) = \frac{e}{r} e^{-\kappa r} + \frac{e\kappa^2}{16p_F^4} \frac{\cos 2p_F r}{r^3} , \qquad (2)$$

so that in the limit $r \to \infty$ the oscillatory component is the predominant component.

We first consider heavy electrons, with a mass M which is quite large. Such electrons become bound at a distance $r_0 \approx (4/\kappa)\ln(4p_F/\kappa)$, which corresponds to the deepest potential well in (2). In this case the bielectron is a spherical rotator with a size r_0 and a binding energy $\varepsilon_0 \approx e\kappa^2/16p_F^4 r_0^3$. The closest excited state corresponds to $l = 1$, so the excitation energy is $\omega = 1/2Mr_0^2$.

An electron localizes within a single well if the level in the neighboring well is more remote than the amplitude for tunneling from well to well. This case is possible if the heavy electron has a large effective mass, $M\kappa^2/16p_F^6 r_0^3 \gg 1$.

In the opposite limit, a bound state spans a group of minima of $V(r)$. We will make use of the quasiperiodic nature of potential (2). Assuming $a = (2M/p_F) \times [E - (e^2/r)e^{\kappa r} - ;(; + 1)/2Mr^2]$ and $q = M\kappa^2 e/16p_F^6 r^3$ to be slowly varying functions of r, we find the following equation for the envelope of the effective-mass method:

$$\varphi'' + (a + \frac{q^2}{2})\varphi = 0. \qquad (3)$$

In the case $l = 0$, the effective potential of Eq. (3) is

$$U(r) = \frac{e}{r} e^{-\kappa r} - \frac{M\kappa^4 e^2}{2^{20} p_F^{10} r^6} . \qquad (4)$$

The potential $U(r)$ has a minimum at the point $r_0 \approx 1/\kappa \ln(2x12^5 p_F^{10}/Me^2\kappa^9)$. The repulsion at $r < r_0 - 1/\kappa$ can be replaced by an infinite wall at the point r_0. The condition for the existence of a bound state then becomes

$$\beta = \frac{Me^2\kappa^2\sqrt{2}}{2^6 p_F^5 r_0^2} \geqslant j_{1/4,\,1}\ , \tag{5}$$

where $j_{1/4,1} = 2.78$ is the first zero of the Bessel function of index 1/4.

If a gas of bielectrons undergoes Bose condensation, we would have $T_c = 3.31N^{2/3}/2M \lesssim \omega$ as the transition temperature, where N is the density of bielectrons. All the arguments above, however, hold if there is no thermal or collisional smearing of the oscillations in (2). From this condition we find the conditions $r_0 \ll v_F/T$, $r_0 \ll l_p$ where l_p is the mean free path.

The interaction between atoms in He^3 is similar to the interaction in (4). We know that in this case the helium liquefies as the temperature is lowered, and then, at significantly lower temperatures, a Cooper pairing with an angular momentum $l = 1$ arises. One might expect an analogous behavior of our electron gas. The condition for liquefaction at absolute zero is the same in order of magnitude as the condition for the formation of a bielectron in (5).

We can show that in our system Cooper pairs with $l = 0$ can form directly from the gas phase if the parameters satisfy a certain relationship. If the density of heavy electrons is low, the interaction of these electrons can be described in terms of an electron scattering amplitude,[5] which is a constant for the scattering of slow particles.[6] It can be determined by solving Eq. (3) under the boundary condition $\varphi(r_0) = 0$:

$$f = \frac{8\pi r_0}{\Gamma^2(\frac{1}{4})}\sqrt{\frac{\beta}{2}}\,\frac{N_{1/4}(\beta)}{J_{1/4}(\beta)}\ . \tag{6}$$

It can be seen from (6) that in the case $J_{1/4,1} < \beta < J_{1/4,1}$ ($J_{1/4,1} = 1.25$ is the first zero of the Neumann function) the scattering amplitude is negative; this negative value corresponds to an effective attraction between particles. If f is sufficiently small ($|f| \lesssim r_0$), the liquefaction of an electron gas does not occur. In this case we can use the standard Cooper-pairing theory with a short-range attraction.[7] For the transition temperature we find

$$T_c = \frac{\gamma\mu}{\pi}\left(\frac{2}{e}\right)^{7/3} e^{-\frac{2\pi^2 v_0}{p_0^2|f|}}\ .$$

Here $\ln\gamma$ Euler's constant, and μ, p_0, and v_0 are respectively the energy, momentum, and Fermi velocity of the heavy electrons. A thermal blurring of the oscillations can lower the transition temperature. The actual value of T_c is on the order of the smaller of the preceding value and a value on the order of v_F/r_0.

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