Quantum theory of closed null supermembranes in four-dimensional space

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A BRST charge is constructed. It is shown that there is no contradiction from the quantum-mechanical standpoint in a theory of null-super-p-branes, including null-supermembranes ($p = 2$), null-superstrings ($p = 1$), and massless superparticles in a 4D space-time.

The problems of a nonphysical dimensionality and of a covariant quantization are central ones in superstring theory.[1] A covariant quantization is complicated by the presence of intertwined constraints of the first and second kinds and by the problem of dealing with the latter in the BFV method.[2,3] In principle, both of these problems could be solved by expanding the original space of dynamic variables of the theory.[4,5] For a covariant separation of constraints, auxiliary harmonic variables[4–6] or twistor variables[7–9] are introduced. The problem of constraints of the second kind is solved by converting these constraints into effective constraints of the first kind, through the use of yet another set of auxiliary fields.[10–12]

In the present letter, we work from these ideas to carry out a covariant quantization of null-super-p-branes in a 4D space. For this purpose we make use of a new twistor formulation[14] of the theory of null-super-p-branes.[13]

After the conversion procedure of Ref. 11 is used, the open algebra of constraints of null-super-p-branes[14] is reduced to an irreducible algebra of rank 2 for effective constraints of the first kind alone,[2)] $\tilde{\tilde{Y}}_{\Lambda}$ and A_f:

$$\{A_f, A_g\} = 0, \quad \{A_f, \tilde{\tilde{Y}}_{\Lambda}\} = 0, \quad \{\tilde{\tilde{Y}}_{\Lambda}, \tilde{\tilde{Y}}_{\Sigma}\} = C^{\Pi}_{\Lambda\Sigma}\tilde{\tilde{Y}}_{\Pi} + \epsilon_{\Lambda\Sigma}, \tag{1}$$

where

$$\epsilon_{\Lambda\Sigma} \equiv \frac{i}{2J}(\delta_{\tilde{\tilde{Y}}_{\Lambda}, \tilde{\tilde{\nabla}}^{-2}}\delta_{\tilde{\tilde{Y}}_{\Sigma}, \tilde{\tilde{\nabla}}^{+2}} - (\Lambda \longleftrightarrow \Sigma))[\tilde{\tilde{D}}^{-i}\tilde{\tilde{D}}_i^{+} + \frac{2}{i}\tilde{\tilde{P}}^{(+1-)}(\tilde{\tilde{\nabla}}^{(0)} + \tilde{\tilde{\nabla}}^{(0)})].$$

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The classical BRST charge[11] Ω corresponding to algebra (1) is $\Omega = \Omega_{min} = \Omega' + A_f C^{\| f}$, where Ω' is given by

$$\Omega' = \; d^p\sigma[C^\Lambda Y_\Lambda^{mod} - 2i\pi^{(+1-)}C_i^+\bar{C}^{-i} - \frac{i}{J}(\pi^{(0)} + \bar{\pi}^{(0)})\tilde{\bar{P}}^{(+1-)} z^{+2}\bar{z}^{-2}$$

$$(\frac{i}{4J}p^{-i}\tilde{\bar{D}}_i^{+} z^{+2}\bar{z}^{-2} + \text{c.c.}) + (\frac{1}{2J}\pi^{(+1-)}\bar{p}_i^+\bar{c}^{-i}z^{+2}\bar{z}^{-2} + \text{c.c.})]. \qquad (2)$$

The modified constraints Y_Λ^{mod} are the same as[14] $\tilde{\tilde{Y}}_\Lambda$, except for the constraints

$$(\nabla^{(0)})^{mod} = \tilde{\tilde{\nabla}}^{(0)} = q_R(Y_\Sigma)P_\Sigma C^\Sigma,$$

$$(\bar{\nabla}^{(0)})^{mod} = \tilde{\tilde{\bar{\nabla}}}^{(0)} - q_L(Y_\Sigma P_\Sigma C^\Sigma.$$

Pairs of canonically conjugate ghost variables (C^Λ, P_Λ) with the brackets $\{C^\Sigma(\vec{\sigma}), P_\Lambda(\vec{\sigma}')\} = -\delta_\Lambda^\Sigma \delta_{\vec{\sigma}\vec{\sigma}}$ and the corresponding constraints $\tilde{\tilde{Y}}_\Lambda$ form the "triads" $(\tilde{\tilde{Y}}_\Lambda; C^\Lambda, P_\Lambda)$:

$$(\tilde{\bar{D}}^{-i}; c_i^+, p^{-i}),\ (\tilde{\bar{D}}_i^{+}; \bar{c}^{-i}, \bar{p}_i^+),\ (\tilde{\bar{P}}^{(+1-)}; \mu^{(-1+)}, \pi^{(+1-)}),\ (\tilde{\tilde{\nabla}}^{(0)}; i\eta^{(0)}; -i\pi^{(0)}),$$

$$(\tilde{\tilde{\bar{\nabla}}}^{(0)}; i\bar{\eta}^{(0)}, -i\bar{\pi}^{(0)}),\ (\tilde{\tilde{\nabla}}^{-2}; z^{+2}, \pi^{-2}),\ (\tilde{\tilde{\bar{\nabla}}}^{+2}; \bar{z}^{-2}, \bar{\pi}^{+2}),\ (\tilde{\bar{T}}_M; \eta^M, \pi_M).$$

We carry out the quantization of this theory in a linearizing gauge, determined by the gauge fermion Ψ_0 and the Hamiltonian $H_{\Psi_0} = (\Psi_0, \Omega)$:

$$\Psi_0 = \gamma/2 \int d^p\vec{\sigma} J^{(-1+)}\pi^{(+1-)}, \qquad (3)$$

$$H_{\Psi_0} = \frac{-\gamma}{2}\int d^p\vec{\sigma} J^{(-1+)}[\tilde{\bar{P}}^{(+1-)} - \pi^{(+1-)}\partial_N\eta^N]$$

$$= \frac{-\gamma}{2}\int d^p\vec{\sigma}[P^m P_m + J^{(-1+)}\pi^{(+1-)}\partial_N\eta^N]. \qquad (4)$$

From the equation of motion $\dot{f} = \{f, H_{\Psi_0}\}$ ($\dot{\bar{x}}^m = 0, \dot{\theta}^\alpha = 0$, etc.) we see that nearly all the phase variables are independent of the time, except for the variables $x^m, P_\alpha^\mp, \mu^{(-1+)}, \pi_M$, which are linear functions of τ and which depend periodically on $\vec{\sigma}$:

$$P_m = P_{0m}(\vec{\sigma}),\ \ v_\alpha^\mp = v_{0\alpha}^\mp(\vec{\sigma}),\ \ \theta_i^\alpha = \theta_{0i}^\alpha(\vec{\sigma}),\ \ \pi_\alpha^i = \pi_{0\alpha}^i(\vec{\sigma}), \ldots \qquad (5)$$

We thus have an indication that an ordering in terms of the operator initial data $\hat{q}_0(\vec{\sigma})\hat{p}_0(\vec{\sigma})$ corresponds to a physical ordering in the quantum picture. We accordingly choose a quantum generalized[3] $\hat{q}\hat{p}$ ordering, and we show that for this ordering the classical equation $\{\Omega, \Omega\} = 0$ becomes the quantum nilpotence condition for the BRST operator, $\hat{\Omega}^2 = 0$, without a generation of anomalous terms. To show this, we note that the overall structure of the quantization of the operator $\hat{\Omega}$ is $\hat{\Omega}$

$= ... + \hat{\mathscr{Q}}\hat{\mathscr{P}} + ... + \hat{\mathscr{Q}}'\hat{\mathscr{P}}' + ...$. Here the $\hat{q}$ chain $\hat{\mathscr{Q}}$ contains only a product of $\hat{q}$ variables ("coordinates"), and the $\hat{p}$ chain $\hat{\mathscr{P}}$ contains only a product of $\hat{p}$ variables ("momenta"): $\hat{\mathscr{Q}} \equiv (q_1)^{A_1}...(q_n)^{A_n}$, $\mathscr{P} = (p_1)^{B_1}...(p_m)^{B_m}$. After expanding the anti commutator $[\hat{\Omega},\hat{\Omega}]_+$, we therefore obtain $[\hat{\Omega},\hat{\Omega}]_+ = ... + [\hat{\mathscr{Q}}\hat{\mathscr{P}},\hat{\mathscr{Q}}'\hat{\mathscr{P}}']_+ = ... + \hat{\mathscr{Q}}[\hat{\mathscr{P}},Q']\hat{\mathscr{P}}' \pm \hat{\mathscr{Q}}'[\hat{\mathscr{Q}},\hat{\mathscr{P}}']\hat{\mathscr{P}} + ...$. Hence the $\hat{q}\hat{p}$ ordering is conserved if it is conserved after the expansion of the (anti-) commutators $[\hat{\mathscr{P}},\hat{Q}']_+$ and $[\hat{Q},\hat{\mathscr{P}}']_\pm$. A *sufficient* condition here is the condition that any pair $(\hat{\mathscr{Q}}, \hat{\mathscr{P}})$ of $\hat{q}$ and $\hat{p}$ chains contained in $\hat{\Omega}$ does not contain more than one pair of canonically conjugate variables $(\tilde{q}^i,\hat{p}_i)$. This condition gives us the condition for distinguishing potentially anomalous terms. Following this condition, we single out from $\hat{\Omega}$ all the nontrivial $\hat{p}$ chains, i.e. $\hat{p}$ chains containing more than one momentum variable. If, for a nontrivial $\hat{p}$ chain $\hat{\mathscr{P}}$ in $\hat{\Omega}$, we find a $\hat{q}$ chain $\hat{\mathscr{Q}}''$ which includes two or more $\hat{q}$ variables which are conjugates of $\hat{p}$ variables forming the given $\hat{p}$ chain $\hat{\mathscr{P}}$, then we need to calculate the (anti-) commutators $[\hat{\mathscr{Q}}'',\hat{\mathscr{P}}]_\pm$ and to distinguish possible anomalous contributions. Proceeding in this manner, we find that the variables $x_{\alpha\dot{\alpha}}$ are included in only one nontrivial $\hat{q}$ chain, $\hat{\eta}^M\partial_M\hat{x}_{\alpha\dot{\alpha}}$, in $\hat{\Omega}$. However, there is no nontrivial $\hat{p}$ chain which includes the product $\hat{\pi}_M\hat{P}^{\dot{\alpha}\alpha}$ in $\hat{\Omega}$. Furthermore, the momentum $\hat{\pi}_M$ forms only a trivial $\hat{p}$ chain which enters the term $\hat{\eta}^N\partial_N\hat{\eta}^M\hat{\pi}_M$ in $\hat{\Omega}$. We thus conclude that the pairs $(\hat{x}_{\alpha\dot{\alpha}},\hat{P}^{\dot{\alpha}\alpha})$ and $(\hat{\eta}^M,\hat{\pi}_M)$ do not contribute to the anomaly, and they can be "canceled out" of the $\hat{q}$ and $\hat{p}$ chains in the discussion below. The same comment applies to canonical pairs of harmonic variables $(\hat{v}^{\alpha\mp},\hat{P}_\alpha^{\mp})$ and $(\hat{\bar{v}}^{\dot{\alpha}\pm},\hat{P}_\alpha^{\mp})$, since the momenta of these pairs, $\hat{P}_\alpha^{\mp},\hat{P}_{\dot{\alpha}}^{\pm}$; enter $\hat{\Omega}$ only in the constraints $\nabla^{(0)}$, $\overline{\nabla}^{(0)}$, $\nabla^{\mp 2},\overline{\nabla}^{\pm 2}$, which form only trivial $\hat{p}$ chains. Allowing for this "cancellation," we find the following potentially anomalous $\hat{\underset{\approx}{p}}$ chains in the operator $\hat{\Omega}$: $\hat{D}^{+i}_{\text{Abel}}(\hat{\bar{D}}_i^-)_{\text{Abel}}$, $\hat{\overline{\overline{D}}}^{-i}(\overline{D}_i^-)_{\text{Abel}}$, $\hat{D}^{+i}_{\text{Abel}}\hat{\bar{\psi}}_i$, $\hat{\overline{\overline{D}}}^{-i}\hat{\bar{p}}_i^+$, $\tilde{\tilde{D}}^{-i}\hat{c}_i^+$, $\hat{\pi}^{(+1-)}\hat{p}^{-i}\hat{c}_i^+$. Their complex conjugates (without the last member of their series) are then nontrivial $\hat{q}$ chains. Going through the procedure of a generalized $\hat{q}\hat{p}$ ordering with the coordinates $\hat{q} = (\hat{x}^m,\hat{\pi}^i_\alpha,\hat{\bar{\theta}}^{\dot{\alpha}i},\hat{\bar{\psi}}^i,\hat{p}^{-i},\hat{\bar{c}}^{-i},...)$ and the momenta $(\hat{p} = (\hat{P}_m,\quad \hat{\theta}^\alpha_i,\hat{\bar{\pi}}_{\dot{\alpha}i},\hat{\bar{\psi}}_i,\hat{c}_i^+,\hat{\bar{p}}_i^+,...)$ which corresponds to the ordering

$$\hat{\tilde{\tilde{d}}}^{-i}\cdot\hat{\bar{\psi}}_i,\hat{\tilde{\tilde{D}}}^{-i}\hat{c}_i^+,\hat{\bar{c}}^{-i}\hat{\tilde{\tilde{D}}}_i^+,\hat{\psi}^i\hat{\tilde{\tilde{D}}}_i^+ ,$$

of the expressions which enter $\hat{\Omega}$, we find that this ordering is conserved in the process of calculating $[\hat{\Omega},\hat{\Omega}]_+$. For example, $[\hat{D}\hat{\bar{D}},\hat{D}\hat{\bar{D}}]_+ \sim [\hat{\pi}\hat{\theta} + \hat{\bar{\theta}}\hat{\bar{\pi}},\hat{\pi}\hat{\theta} + \hat{\bar{\theta}}\hat{\bar{\pi}}]_+ \Rightarrow \hat{\pi}\hat{\theta} + \bar{\theta}\hat{\bar{\pi}}$. Consequently a generalized $\hat{q}\hat{p}$ ordering is free of anomalies and must be used in going over to the quantum picture in the theory of null-super-p-branes.

The transition from a $\hat{q}\hat{p}$ ordering to an ordering in terms of the operator initial data $(\hat{q}_0(\vec{\sigma}),\hat{p}_0(\vec{\sigma}))$ does not introduce any fundamental changes, as can be seen directly and easily by examining the solutions of the equations of motion for the variables $\hat{q}$, $\hat{p}$. The reason is that for the variables $\hat{q}$, $\hat{p}$, which are linear functions of the time, nonlinear combinations of the initial data for τ do not generate any new nontrivial $\hat{p}$ chains in $\hat{\Omega}$. For example, in the solution for x^m the nonlinear increment is a trivial $\hat{p}$ chain (corresponding comments apply to the variables $P_\alpha^{\pm}$, $\mu^{(-1+)}$, $\pi^{(+1-)}$):

$$x^m(\tau,\vec{\sigma}) = x_0^m(\vec{\sigma}) + \tau\gamma\left[P_0^m(\vec{\sigma}) + \frac{(v_0^+\sigma^m\bar{v}_0^-)}{\sqrt{(1+\Xi_0)(1+\bar{\Xi}_0)}}\pi_0^{(+1-)}\partial_N\eta_0^N\right].$$

We have thus proved that a generalized $\hat{q}_0\hat{p}_0$ ordering does not lead to any anomalous increments, so a quantum BRST charge $\widehat{\Omega}$ is nilpotent: $\widehat{\Omega}^2 = 0$. The theory of null-super-p-branes is therefore a noncontradictory quantum theory of extended p-dimensional entities in a realistic 4D space-time.[4)]

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[2)] We do not have space here to reproduce the explicit expressions for these constraints $\tilde{\tilde{Y}}_\Lambda, A_f$ and other definitions; all are given in Refs. 14.

[3)] The word "generalized" here means that certain momentum variables may be part of a $\hat{q}$ set, and that, correspondingly, there may be coordinate variables in a $\hat{p}$ set.

[4)] We would like to point out a new mechanism for the generation of a tension $\sim 1/\alpha'$ for null-superstrings: through the addition to their action of a Wess–Zumino term, which *breaks* the fermion K symmetry of the action of the null-superstrings:[13] $S_1 \Rightarrow \tilde{S}_1 = -\frac{1}{2}\int d\tau\, d\sigma \cdot \{\rho^\mu v^{-\alpha}\bar{v}^{+\dot{\alpha}} w_{\mu\alpha\dot{\alpha}} + (i/\alpha')\varepsilon^{\mu\nu}(\partial_\nu\theta^\alpha\bar{\theta}^{\dot{\alpha}} - \theta^\alpha\partial_\nu\bar{\theta}^{\dot{\alpha}})w_{\mu\alpha\dot{\alpha}}\}$.

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