Channeling, collimation, and radiation of relativistic electrons in ultrastrong nonuniform optical fields

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Relativistic electron beams can be channeled and collimated in intense interference optical fields. Certain aspects of the radiation by a channeled electron are discussed.

1. This letter reports a theoretical study of the interaction of relativistic electron beams with ultrastrong, spatially nonuniform, coherent optical fields. It is predicted that electrons can be channeled, i.e., that there can be a regime of finite motion of electrons near the nodes of an interference optical field. The critical channeling angles are derived. The particular features of the radiation by electrons during channeling are studied. It is shown that relativistic beams can be collimated.

A ponderomotive force proportional to the gradient of the field intensity acts on a free electron, atom, or microscopic particle in a spatially nonuniform electromagnetic field, as is well known. This force underlies several methods for controlling the motion of particles,[1] performs a bunching of particles in free-electron lasers,[2] has a significant effect on the ionization spectra of atoms,[3] etc. The motions caused by this force in fields of moderate intensity are slow in comparison with the oscillations of the electron at the optical frequency. In intense interference laser fields, however, the gradient force may substantially change the very nature of the motion of the particle and thereby lead to a qualitative change in its high-frequency response. One such effect, which leads to a qualitative change in the nature of the particle-field interaction, is the channeling of relativistic particles in an interference laser field.

2. We consider the very simple case of an interference field which is a superposition of two plane waves. We assume that the field is a transverse electric field. The electric and magnetic fields are then given by (Fig. 1)

$$\vec{E} = \vec{E}_0 \cos \kappa_\perp x \cos(\kappa_\| z - \omega t),$$

$$\vec{H} = \vec{H}_\perp \cos \kappa_\perp x \cos(\kappa_\| z - \omega t) - \vec{H}_\| \sin \kappa_\perp x \sin(\kappa_\| z - \omega t), \qquad (1)$$

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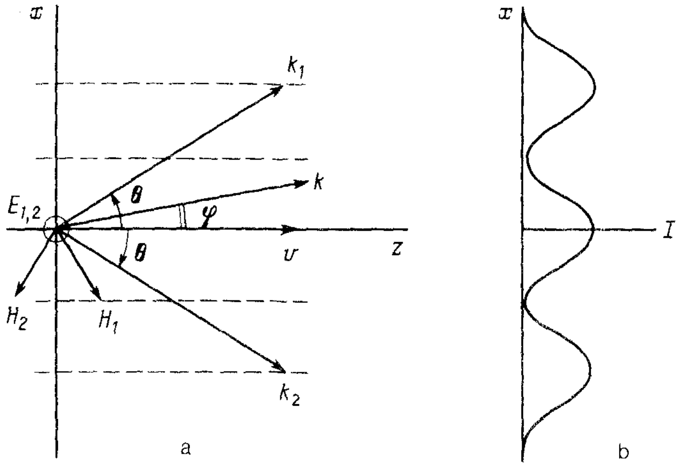

FIG. 1. a—Relative arrangement of the wave vectors of the interfering waves ($\vec{k}_1$ and $\vec{k}_2$), of the electron velocity $\vec{v}$ ($|\vec{v}|\approx c$), and of the wave vector of the wave emitted by the electron ($\vec{k}$) (the dashed lines show the nodes and antinodes of the interference field); b—intensity of the interference field (the ponderomotive potential) as a function of the transverse coordinate.

where $\kappa_\perp = \kappa_x = \kappa \sin\theta$, $\kappa_\| = \kappa_z = \kappa\cos\theta$, $\kappa = \omega/c$, $H_\perp = -E_0\cos\theta$, and $H_\| = E_0\sin\theta$. An electron is moving along the z axis, parallel to the antinodes of the interference field, so the longitudinal coordinate of the electron is given in a first approximation by

$$z(t) = z_0 + vt, \tag{2}$$

where $z_0 = z(t=0)$. The equations for the transverse motion of the electron are

$$\ddot{x} = \frac{eE_0}{m}\frac{\dot{y}}{c}\sin\theta\sin\kappa_\perp x\sin[\kappa_\| z_0 - \omega(1-\beta\cos\theta)t],$$

$$\ddot{y} = -\frac{eE_0}{m}\{(1-\beta\cos\theta)\cos\kappa_\perp x\cos[\kappa_\| z_0 - \omega(1-\beta\cos\theta)t] + \frac{\dot{x}}{c}\sin\theta\sin\kappa_\perp x\sin[\kappa_\| z_0 - \omega(1-\beta\cos\theta)t]\}. \tag{3}$$

Integrating the second equation in (3), and substituting the result into the first, we find

$$\ddot{\xi} + \Omega^2(1-\cos[2\omega(1-\beta\cos\theta)t])\sin\xi = 0, \tag{4}$$

where

$$\xi = 2\kappa_\perp x - \pi, \quad \Omega = \frac{eE_0\sin\theta}{\sqrt{2}mc}. \tag{5}$$

It follows from (4) that under the condition $\Omega \ll \omega(1-\beta\cos\theta)$ the motion of the particle is a superposition of high- and low-frequency oscillations. The amplitude of the high-frequency oscillations is given by

$$a_\omega = a_\Omega[\Omega/\omega(1-\beta\cos\theta)]^2,$$

where $a_\Omega = 1/2\kappa_\perp$ is the amplitude of the low-frequency oscillations in the potential

$$U(x) = U_0(1+\cos 2\kappa_\perp x)/2, \quad U_0 = e^2E_0^2/4m\omega^2. \tag{6}$$

Under the condition $a_\omega \ll a_\Omega$ with $\Omega < \omega(1-\beta\cos\theta)$, and if the energy of the transverse motion of the electron is below the barrier height U_0, the electrons thus execute bounded oscillations between two antinodes of the interference field. It is therefore a straightforward matter to derive the following expression for the critical channeling angle:

$$\eta_c = \frac{p_{\perp max}}{p} = \sqrt{\frac{2mU_0}{p^2}} = \frac{eE_0}{\sqrt{2}m\omega c}. \tag{7}$$

Assuming $\lambda = 1\ \mu$m, and expressing E_0 in (6) in terms of the intensity I, we find

$$U_0(\text{eV}) = (10^{-13}/\gamma)I(\text{W/cm}^2),$$

i.e., $U_0 = 1$ eV with $\gamma = 10^2$ and $I = 10^{15}$ W/cm^2.

3. The oscillations of the particle along the x axis give rise to Doppler branches of the radiation, whose frequency-angular spectrum is given by

$$\nu_{2n,m} = \frac{2\omega(1-\beta\cos\theta)n \pm \Omega_m}{1-\beta\cos\varphi}, \quad n,m = 0,\ 1,\ 2,..., \tag{8}$$

where φ is the angle between the z axis and the radiation direction.

A case of particular interest is that in which the condition for a parametric resonance holds:

$$\Omega = \omega(1-\beta\cos\theta).$$

In this case the amplitude of the transverse oscillations of the electron increases. According to (3), the oscillations of the electron along the y axis give rise to a Doppler radiation branch with the spectrum

$$\nu_{n,m} = \frac{\omega(1-\beta\cos\theta)n \pm \Omega m}{1-\beta\cos\varphi}, \quad n,m = 0,\ 1,\ 2,\dots\ . \tag{9}$$

4. The radiation by a particle moving in potential (6) results in a radiative damping of the oscillation amplitude if the time scale of the interaction of the particle with the interference field (t_{int}) is much longer than the radiative damping time T_1: $t_{\text{int}} \gg T_1$. Under this inequality, relativistic beams will be collimated.

5. In summary, this analysis shows that an intense interference field changes the behavior of a relativistic particle in a qualitative way when the particle is moving parallel to the interference planes. In this case the motion of the particle along the transverse coordinates becomes finite under certain conditions, and new Doppler branches appear in the spectrum radiated by the particle. The frequencies of these new branches differ from the frequency of the controlling interference field.

[1] I. R. Shen, *Principles of Nonlinear Optics*, Nauka, Moscow, 1989.

[2] E. G. Bessonov and A. V. Vinogradov, Usp. Fiz. Nauk **159**, 143 (1989) [Sov. Phys. Usp. **32**, 806 (1989)].

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