Phonon superconductivity mechanism in strongly correlated systems

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(Submitted 16 November 1990)
Pis'ma Zh. Eksp. Teor. Fiz. **53**, No. 1, 21–23 (10 January 1991)

The renormalization of T_c for the electron–phonon pairing mechanism, with a formally exact allowance of the electron–electron coupling in the normal phase, has been calculated. The electron–phonon mechanism was found to be strongly suppressed in the marginal Fermi-liquid state and it does not lead to superconductivity near the Mott–Hubbard transition.

The nature of the electron spectrum of high-T_c superconductors and the nature of superconductivity are now being vigorously discussed. Two generally different problems are discussed: the applicability of the Fermi-liquid model (or even the band theory) in the normal phase and the pairing mechanism (magnetic, phonon, etc.). Since a small isotopic effect (which is particularly noticeable in $La_{2-x}Sr_xCuO_4$) has definitely been observed in high-T_c superconductors,[1] the electron–phonon mechanism clearly contributes to T_c, although possibly it may not be the principal contribution. At the same time, the experimental data on the properties of the normal phase of a high-T_c superconductor suggest that there is an electron–electron correlation right to the point at which the Fermi-liquid model breaks down. There are basically different descriptions of this state (see Refs. 2 and 3, for example). It is therefore of interest to analyze the electron–phonon mechanism for different models of strongly correlated systems.

Ignoring the effect of the electron–electron component in the anomalous part of the Green's function, we have an expression for the contribution of the electron–phonon coupling to the self-energy matrix part (we are analyzing the singlet pairing):

$$\hat{\Sigma}_{ph}(\vec{k}, ip_n) = T \sum_{\vec{q}, i\omega_m} V_{\vec{k}\vec{q}}(i\omega_m)\hat{\tau}_3 \hat{G}(\vec{q}, ip_n - i\omega_m)\hat{\tau}_3, \tag{1}$$

where $\hat{G} = [\hat{g}^{-1} - \hat{\Sigma}_{ph}]^{-1}$ is the electronic Green's function, $\hat{g}$ is the Green's function in the normal phase, with an exact allowance for the electron–electron coupling,

$$V_{\vec{k}\vec{q}}(i\omega_n) = 2\int_0^\infty d\Omega \sum_j |M_{\vec{k}\vec{q}j}|^2 B_{\vec{k}-\vec{q},j}(\Omega)\frac{\Omega}{\omega_n^2+\Omega^2} \tag{2}$$

is the effective interaction potential, $M_{\vec{k}\vec{q}j}$ is the matrix element of the electron–phonon coupling, $B_{\vec{q}j}(\Omega)$ is the spectral phonon density, j is the branch index, $\hat{\tau}$ are the Pauli matrices, $\omega_m = 2\pi mT$, and $p_n = (2n+1)\pi T$ are the Matsubara frequencies.

We seek the solution of Eq. (1) in the form

$$\hat{\Sigma}_{ph}(\vec{k}, ip_n) = iP_{\vec{k}}(ip_n)[1 - Z_{\vec{k}}(ip_n)]\hat{\tau}_0 + \chi_{\vec{k}}(ip_n)\hat{\tau}_3 + \phi_{\vec{k}}(ip_n)\hat{\tau}_1, \tag{3}$$

$$iP_{\vec{k}}(ip_n) = ip_n - \frac{1}{2}[\Sigma_e(\vec{k}, ip_n) - \Sigma_e(-\vec{k}, -ip_n)],$$

where Σ_e is the self-energy part of the normal phase, which is attributable to the electron–electron coupling. Substituting (3) in (1), we find the Eliashberg equation in the form

$$\chi_{\vec{k}}(ip_n) = -T\sum_{\vec{q},ip_m} V_{\vec{k}\vec{q}}(ip_n - ip_m)\epsilon_{\vec{q}}(ip_m)\Psi_{\vec{q}}(ip_m),$$

$$\phi_{\vec{k}}(ip_n) = T\sum_{\vec{q},ip_m} V_{\vec{k}\vec{q}}(ip_n - ip_m)\phi_{\vec{q}}(ip_m)\Psi_{\vec{q}}(ip_m), \tag{4}$$

$$\Psi_{\vec{q}}^{-1}(ip_m) \equiv P_{\vec{q}}^2(ip_m)Z_{\vec{q}}^2(ip_m) + \epsilon_{\vec{q}}^2(ip_m) + \phi_{\vec{q}}^2(ip_m),$$

where $\epsilon_{\vec{q}}(ip_n) = \epsilon_{\vec{q}} + \frac{1}{2}[\Sigma_e(\vec{k},ip_n) + \Sigma_e(-\vec{k}, -ip_n)] + \chi_{\vec{q}}(ip_n)$, and $\epsilon_{\vec{q}}$ is the band energy. In the approximation of the weak electron–phonon coupling we set[4]

$$V_{\vec{k}\vec{q}}(ip_n - ip_m) = [\lambda/N_0(E_F)]\theta(\omega_D - |p_n|)\theta(\omega_D - |p_m|), \tag{5}$$

where λ is the dimensionless constant of the electron–phonon coupling, $N_0(E)$ is the bare density of states, and ω_D is the Debye frequency. We then have $Z_{\vec{k}}(ip_n) = 1$, and $\chi_{\vec{k}}(ip_n) = 0$. We will also ignore the $\vec{k}$ dependence $\Sigma_e(\vec{k},ip_n)$, which is valid for the most important models of the strongly correlated systems. We will then obtain from (4) an equation for the critical temperature T_c

$$1 = \frac{\lambda T_c}{N_0(E_F)}\sum_{|p_n|<\omega_D}\frac{ip_n}{iP(ip_n)}\int_{-\infty}^{+\infty} d\epsilon \frac{N(\epsilon)}{\epsilon^2 + p_n^2}, \tag{6}$$

where $N(\epsilon) = (-1/\pi)\Sigma \mathrm{Im} g_{11}(\vec{k},\epsilon)$ is the density of states in the normal phase of a strongly correlated system, with allowance for the electron–electron renormalizations. If the characteristic electron energies are higher than ω_D [adiabatic approximation used in the derivation of (1)], and $N(E_F)$ is a finite value, we then have from (6) the expression

$$T_c = 1,13\omega_D e^{-1/\Lambda}, \ \ \Lambda = \lambda a N(E_F)/N_0(E_F), \ \ a = \left(1 - \frac{\partial \Sigma_e(E)}{\partial E}|_{E=E_F}\right)^{-1}. \quad (7)$$

If the theory of Fermi liquid is valid, then $N(E_F) = N_0(E_F)/a$, and $\Lambda = \lambda$ even in the limit $a \to 0$ (the "heavy-fermion" case). There is therefore no renormalization of electron–phonon coupling, at least if $a < \omega_D/E_F$ (adiabaticity condition).

If the theory of marginal Fermi liquid[2] $N(E_F) \simeq N_0(E_F)$, $P(\epsilon) \sim \epsilon \ln|\epsilon/\omega_c|$ ($|\epsilon| < \omega_c$), where $\omega_c \sim E_F$ is the characteristic energy of the charge fluctuations), and we have from (6)

$$T_c \simeq \omega_D \exp\left(-\left[\exp\left(\frac{N_0(E_F)}{\lambda N(E_F)}\right) - 1\right] \ln \frac{\omega_c}{\omega_D}\right), \quad (8)$$

which shows that T_c is strongly suppressed for small values of λ. In contrast, in the Anderson theory[3] the effective $N(\epsilon)$ diverges as $\ln^2\epsilon$ as $\epsilon \to 0$ because of the "infrared catastrophe," which accounts for the increase in T_c

$$T_c \sim \omega_D \exp(-\text{const}/\lambda^{1/3}).$$

The same situation holds if the manner in which $N(\epsilon)$ behaves, $N(\epsilon) \sim \ln^2\epsilon$, is due to the van Hove singularities.[5]

A description of the metal–insulator transition in the Hubbard model shows that there is a strong damping of the electronic states on E_F in the Hubbard-III approximation, and that the Fermi-liquid theory clearly does not hold. Near the critical value of the Hubbard repulsion $U_c = (6\mu_2)^{1/2}$ (μ_n is the moment of the bare density of states of order n) we have the expression[7]

$$E - \Sigma_e(E) \simeq \alpha(U) + \beta(U)E, \ \ \alpha(U) \cong (4M/(U^2 - U_c^2))^{1/2}, \quad (9)$$

$$\beta(U) = -\alpha^4(U)/6M, \ \ M = \mu_4 - \mu_2^2.$$

For $U \gtrsim U_c$ (the insulating phase) α is valid, $N(E_F) = 0$, and there is no superconductivity. For $U \lesssim U_c$, $N(E_F) \sim 1/|\alpha| \sim (U_c - U)^{1/2}$. In this case, $a \simeq \beta^{-1} < 0$, in contrast with the Fermi liquid, and Eq. (6) has no solutions. If, on the other hand, the pairing still occurs (due to the electron–electron coupling, for example), then the electron–phonon contribution will inhibit it. The corresponding coupling constant

$$\Lambda \sim \lambda(|\alpha|\beta)^{-1} \sim -(U_c - U)^{5/2} \quad (10)$$

will, however, be small near the metal–insulator transition. The change in the sign of $\partial \operatorname{Re}\Sigma(E)/\partial E$ because of a strong damping is similar to the anomalous dispersion in optics.

The effect of the electron–phonon mechanism on the onset of superconductivity in strongly correlated systems thus depends on the perceived behavior of the normal phase. Remaining constant for the Fermi liquid, this mechanism is suppressed in the theory[2] and is enhanced (as any other mechanism) in the treatment used in Ref. 3. In the Hubbard-III approximation the electron–phonon coupling leads to an effective electron–electron repulsion.

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Translated by S. J. Amoretty