Electron paramagnetism of a ring

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An exact solution of the spectrum of electrons localized on a ring in a transverse magnetic field, which depends on the electron number parity, has been found in the strong-coupling approximation. The ground state with a magnetic field flux has been shown to exist on the ring.

The probability that a ground state with a nonzero orbital current can now exist is of enormous interest. Such states are states with broken fundamental symmetries T, P, and C (Refs. 1–7). It was shown in Refs. 6 and 7 that the energy of electrons at the lattice in a magnetic field is lower than the energy of free electrons without a field. This shows that the state with a flux exists and that it is the ground state of electrons at the lattice. There is usually the objection, however, that when the magnetic field energy is taken into account, the total energy is still higher than that of free fermions. We will show here that for a simple two-dimensional lattice system—a ring on which there are nodes with electrons, the electron energy, along with the field energy, is lower than the energy of free electrons on the ring.

For simplicity, we will ignore the spin. The Hamiltonian of the spinless fermions on the ring in a magnetic field will then have the form

$$H = -\sum_{<i,j>}^{N} t_{ij} a_i^+ a_j, \tag{1}$$

where a_i^+ and a_j are the operators for creating and annihilating fermions, the sum $\langle i,j \rangle$ is taken only over the nearest neighbors, and N is the number of sites on the ring. In the symmetrical gauge the diagonalization of the Hamiltonian under consideration reduces to the diagonalization of the $N \times N$ matrix.

$$A = -\begin{pmatrix} 0 & p & 0 & 0 & 0 & \dots & 0 & p^{-1} \\ p^{-1} & 0 & p & 0 & 0 & \dots & 0 & 0 \\ 0 & p^{-1} & 0 & p & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & 0 & p \\ p & 0 & \dots & \dots & \dots & 0 & p^{-1} & 0 \end{pmatrix} \times t, \tag{2}$$

where $p = \exp(i2\pi f/N)$; $N \geqslant 3$ and f is the magnetic flux in the ring in units of the flux quantum. By using the induction method it can be easily shown that the eigenvalues of this matrix are

$$E_{Nm} = -2t \cos\left(\frac{2\pi}{N} f + \frac{2\pi m}{N}\right); \tag{3}$$

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here $0 \leqslant f \leqslant 1$ and $m = 0, 1, 2, \ldots N-1$.

Placing the fermions in the lower-lying levels, we can easily calculate the ground state energy. This energy depends strongly on whether an even or an odd number of electrons are in the ring. If the system has an even number of electrons, the ground state energy of $2M$ electrons is given by

$$E_{gr}^{c} = -2t\left[\alpha\cos\left(\frac{2\pi f}{N}\right) + \beta\sin\left(\frac{2\pi f}{n}\right)\right], \tag{4}$$

where $\beta = \sin(2\pi M/N)$ and $\alpha = \mathrm{cotg}(\pi/N)\beta$. As can be seen from this expression, the energy depends linearly on the flux and in this case decreases with increasing f from zero. Since the field energy always depends quadratically on the flux, the total energy of the system clearly decreases when the field is applied. If the field energy is given by Cf^2, where C is a constant, the equilibrium magnetic field will be

$$f_{sp} = \frac{2\pi t\beta}{CN}. \tag{5}$$

As can be seen from this equation, the flux is small in the parameter $1/N$. The magnetization, which can easily be determined from Eq. (3), is also small in this parameter. The energy gain due to the appearance of an effective magnetic field is nonappreciable: on the order of $1/N^2$. This effect occurs because all the levels, except the upper and lower levels, are doubly degenerate in the absence of a field, while the degeneracy is lifted upon application of the field.

Let us examine the results which we obtained. In the case of odd filling of the electrons, their energy is a periodic function with a period $f = 1$. The energy reaches a minimum value at integer values of the flux $f = \ldots, -2, -1, 0, +1, +2, \ldots$. With half-integer values of the flux, the electron energy exhibits a sharp single peak corresponding to the phase transition to a state with a new quantum number of the total flux, which is equal to the sum of the external and diamagnetic electron fluxes.

When the ring has an even number of electrons, the energy of these electrons is a periodic function of the field with a period $f = 1$. The behavior of this function is similar to that of the energy of the system with an odd number of electrons, but one which is shifted by a half-period. In other words, when the magnetic flux has integer values, the energy, a function of the magnetic flux, has sharp maxima, and when the flux has half-integer values, the energy has smooth minima. These results obtained by us can be used to explain the experiments on the persistent currents in the mesoscopic rings.[8] In Ref. 8 the magnetic moment with a half-integer period in systems consisting of copper rings was found to oscillate. Although we found that spontaneous magnetization is small in the parameter $1/N$, this small value in an actual situation is either nullified or can be nullified by the transverse degeneracy, since a copper ring has a very large number of chains consisting of N nodes. If half the rings have an even number of electrons, on the average, and the other half of the rings have an odd number, then the ground state energy of such a system is a periodic function, but one which has a half-period. It thus follows that both the magnetic-moment oscillations and the magnetic-susceptibility oscillations have a half-period.

The spontaneous electron magnetization described here can be observed in rings consisting of quantum points which can be fabricated because of the advances of modern technology.[9]

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