Impurity luminescence from a Laughlin 2D electron gas

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The spectrum of the impurity recombination radiation from a 2D electron gas is calculated in the spherical Haldane model for a filling factor $\nu = 1/3$. The spectrum depends critically on the angular momentum of the system in its initial state. A change in the value of the angular momentum may cause abrupt changes in the emission spectrum as a function of the parameters of the system. The factors which determine the intensity of the line corresponding to the transition to the ground state of the system are studied. The position and intensity of the Auger satellites are also studied.

The electron–electron interaction in a quasi-two-dimensional (2D) electron gas in a high magnetic field leads to several specific effects, the most noteworthy being the fractional quantum Hall effect.[1] The incompressible quantum fluid[2] responsible for this transport effect has recently been the subject of active research by optical methods, which have also been used to study other 2D phases. Both the trapping of carriers by impurity centers near the confinement region[3,4] and the recombination of free carriers in this region[5,6] are under study. Significant progress has been achieved in the analysis of such spectra, but a more comprehensive interpretation of these spectra can be achieved only through a quantitative comparison with theory. The difficulty is that the problem lacks a small parameter: All the competing quantities are at the scale of the Coulomb energy $\epsilon_c \equiv e^2/(\epsilon l)$, where ϵ is the dielectric constant, and l the magnetic length. For this reason, perturbation theory cannot be used, and the arsenal of methods which can be brought to bear is extremely limited. The first theoretical studies[7–9] discussed the appearance in the luminescence spectrum of satellites as a result of Auger processes in the electron gas. An exactly solvable model of three electrons coupled by a short-range interaction was studied in Ref. 7. The position and intensity of the satellites were studied as a function of the parameters of the model. The results of a numerical solution of a four-electron model in a rectangular geometry were briefly reported in Ref. 8. The same problem was discussed in Ref. 9 in a phenomenological way in a single-mode model.

In this letter we are reporting a numerical solution of the problem of the trapping of an electron from a 2D fluid by a neutral or attractive Coulomb center at an electron density corresponding to a filling factor $\nu = 1/3$. There are six electrons in the initial state and five in the final state. We study the overall spectrum: the positions of the satellites and the intensity distribution. We find the optimum conditions for extracting information on the parameters of the spectrum of elementary excitations from such luminescence spectra. The ν dependence of the spectrum is described separately.

The calculation was carried out in spherical geometry,[10] which is convenient in spectroscopic applications, since the continuous rotation group and the associated

selection rules are retained. Spin-oriented states exclusively were considered. The customary calculation approach was taken.[10-12] The distance between points on the sphere was assumed to be equal to the length of the chord. In contrast with earlier work on the effective impurities on a 2D gas,[13,14] the displacement (h) of the Coulomb center from the confinement plane was simulated not through the introduction of an effective charge Z^* but through a displacement of the center from the north pole of the Haldane sphere ($\theta = 0$), over a distance h^* toward the center of the sphere. The correspondence between h and h^* can be determined from the condition that the potential created by the Coulomb center at the center of the plane is equal to the potential difference between the north and south poles of the sphere:

$$h^* = (R + h) - (R^2 + h^2)^{1/2}. \tag{1}$$

Here $R = lS^{1/2}$ is the radius of the sphere, and $2S$ is a parameter of the theory, equal to the total magnetic flux through the sphere (in units of the flux quantum). This method is being used to describe the impurity centers because it describes not only the weakening of the Coulomb field as the center moves away from the plane but also the smoother spatial distribution of this field. If we introduce the parameter $\alpha = 1 - h^*/R$, the value $\alpha = 0.75$ corresponds approximately to Si MOSFETs, where $h \approx 60$ Å (Ref. 15), while values $\alpha = 0.4$–0.5 correspond to the situation in GaAs/AlGaAs heterostructures, with $h \approx 200$–250 Å (Ref. 4). In each case we have $l \approx 100$ Å. We assume that the inequalities $l, h \gg r_i$ hold, where r_i is the radius of the impurity center, a is the lattice constant, and we are using the effective-mass approximation. The interaction of the neutral center with the 2D gas is omitted, and the potential of the charged center is assumed to be a Coulomb potential. In the basis of eigenfunctions of the angular momentum operator, $\psi_m \propto [\cos(\theta/2)]^{S+m}[\sin(\theta/2)]^{S-m}\exp(im\varphi)$, where θ and φ are polar coordinates, the energy matrix elements of an electron in the field of a Coulomb center are

$$V_m(h^*) = -(Ze^2/R)F(1/2, S+m+1, 2(S+1); \\ (1-h^*/R)/(1-h^*/2R)^2)/(2-h^*/R), \tag{2}$$

where $F(\alpha,\beta,\gamma,z)$ is the hypergeometric function.

The probability for radiative recombination satisfies the proportionality

$$w_{MM'} = 2\pi(N/R^2)\left|\int \ldots \int d\vec{\omega}_1 \ldots d\vec{\omega}_{N-1}\Phi^*_M(\vec{\omega}_1\vec{\omega}_2 \ldots \vec{\omega}_N = \vec{\omega}_0)\Psi_{M'}(\vec{\omega}_1 \ldots \vec{\omega}_{N-1})\right|^2, \tag{3}$$

where N is the number of electrons in the initial state, Φ_M and $\Phi_{M'}$ are normalized antisymmetric wave functions of the initial and final states, M and M' are projections of the angular momentum, and the integration over $\vec{\omega}_j$ is extended over a unit sphere. It is assumed, by virtue of the inequality $r_i \ll h$ and the exponentially small value of the overlap integral, that the recombination occurs at the north pole, $\vec{\omega}_0$: the point on the sphere closest to the impurity center. It can be seen from the expression for ψ_m that recombination occurs only from the $m = S$ orbital, so we find that the following selection rule from (3):

$$M = M' + S. \tag{4}$$

We first consider an impurity center with $Z = 1$; after recombination, this center is neutral ($Z = 0$). The number of electrons in the final state is $N = 5$. This number corresponds to an incompressible fluid with $\nu = 1/3$ at $S = 3(N-1)/2 = 6$. Adopting this value of S, we can study the relationship between the luminescence spectrum and the spectrum of excitations of a free 2D fluid. The results are shown in Fig. 1 for

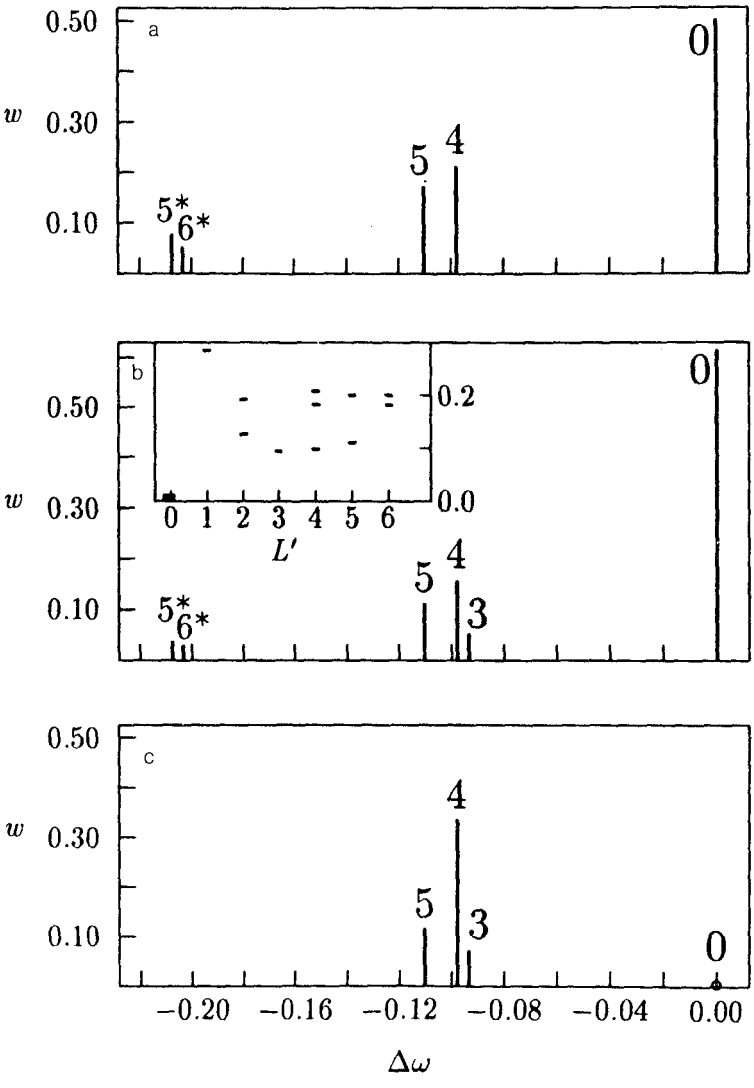

FIG. 1. Emission spectrum accompanying the trapping of an electron by a positively charged center. In the initial state there are $N = 6$ electrons; in the final state there are $N = 5$, and an incompressible fluid forms. The energies are reckoned from the transition to the ground level of the system. The value of L' for the final level is shown for each band. Transitions to levels which are not the lowest levels for the given L' are marked with an asterisk. a—$\alpha = 1$; b—$\alpha = 0.5$; c—$\alpha = 0.3$. The inset shows the lower part of the spectrum of elementary excitations in the final state.

three values of α. For the values $\alpha = 0$ and 0.5 in the initial state ($N = 6$), the ground level is a level with $M = S = 6$ (the gaps are 0.050 and 0.034, respectively; all energies are expressed in units of ϵ_c below). According to (4), transitions occur to a level with $M' = 0$ and with various values of the total angular momentum L'. The transition to the Laughlin state ($L' = 0$) is the most intense. Figures 1a and 1b, show the strongest satellites (with an intensity $\geqslant 5\%$ of the intensity of the fundamental band). The most intense of these satellites correspond to states near the minimum of the roton branch [$kl = L(l/R) \approx 1.6$ with $L = 4$], i.e., to a roton gap Δ_r (Ref. 16). Note that excitations with a higher energy also contribute significantly. At $L' = 6$, the intensity of the satellite corresponding to the lower level is $\sim 10^{-3}$, and the contribution of the following level is dominant. Since the probability w, given by (3), is not expressed in terms of the electron density, there is no general reason for a domination of roton satellites. Nevertheless, this is what we find, everywhere except in the low-frequency part of the emission spectrum.

At $\alpha \leqslant \alpha_{cr} \approx 0.38$ the $M = 3$ level is the lowest level in the initial state (a term crossing upon a variation of α was noted in Ref. 14). In the final state we thus have $M' = -3$; i.e., transitions can occur only to levels with $L' \geqslant 3$. Figure 1c shows the emission spectrum for $\alpha = 0.3$. We see that the most intense band has a significant low-frequency shift (although the magnitude of this shift, ≈ 0.10, is numerically close to the size of the gap for the formation of a quasiparticle–hole pair, it is totally unrelated to this gap). This sharp change in the spectrum occurs as a transition as α is varied, at $\alpha = \alpha_{cr}$, but it can also be controlled by other parameters (ν, the temperature T, etc.). If there are recombination centers with different values of α, there may be superposition of two spectra. All the intense bands correspond to the roton branch.

Figure 2 shows the emission spectrum for the same system, but for trapping by a neutral center. The initial level is a degenerate level with $L = 3$ ($-3 \leqslant M \leqslant 3$). Transitions to levels $-9 \leqslant M' \leqslant -3$ are therefore allowed. The final state corresponds to an incompressible fluid perturbed by the Coulomb potential of the impurity. At $\alpha = 1$, the level with $M' = -8$ is the lowest level; at $\alpha \leqslant 0.36$, the level with $M' = 0$ is the lowest. The emission spectrum is seen to consist of many bands of comparable intensity. These bands correspond to different values of M'. Most of the bands lie within a region of width Δ_r, reflecting the absence of a gap at $\alpha = 1$ (Ref. 14). At $\alpha = 0.3$, the ground level is $M' = 0$, and a transition to it is forbidden. The most intense bands ($M' = -4$ and $M' = -8$) in the spectra of Figs. 2a and 2b, occur in the opposite order. The insets show the lower part of the spectrum of elementary excitations, which is quite different from the spectrum in Fig. 1b (the inset). The lower part of the spectrum is a "quasiroton" branch. The most intense satellites in Fig 2a and the intense long-wavelength bands in Fig. 2b correspond to this branch. However, the low-frequency part of the emission spectrum, which corresponds to transitions to higher-lying levels, with an energy at the scale of Δ_r, also has a significant intensity (particularly in Fig. 2b).

The emission spectrum of the same system at $S = 7.5$ (a Laughlin phase in the initial state, but not in the final state) is dominated by a band corresponding to a transition to the ground level ($M' = -7.5$); the intensity of the satellites is $\approx 2.5\%$. This result shows that the rich spectrum of satellites in Figs. 2a and 2b, is a conse-

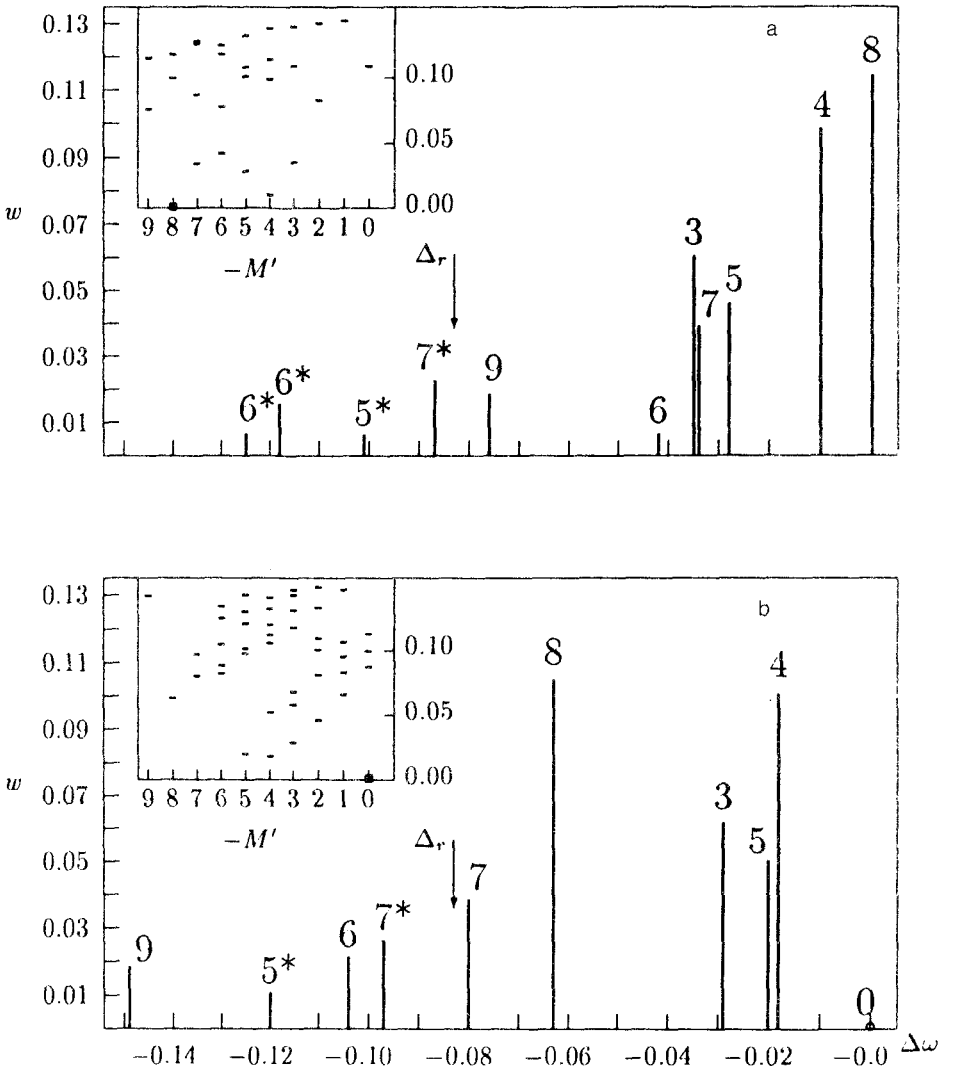

FIG. 2. Emission spectrum accompanying the trapping of an electron by a neutral center. The value of $(-M')$ for the final level is shown for each band. It is assumed that all the mutually degenerate sublevels of the ground state are filled identically. The situation is otherwise as in Fig. 1. a—$\alpha = 1$; b—$\alpha = 0.3$. The inset shows the lower part of the spectrum of elementary excitations in the final state.

quence of various excited states of the "extra" electron which was present in the initial state.

In the limit $N \to \infty$, most of the bands merge into a continuum.

It can be seen from these results that the emission spectrum is very sensitive to several properties of the system: the charge of the impurity center, the distance from this center to the confinement plane, and the presence of quasiparticles in the initial

state. The satellite intensity is high if the impurity is neutral in the final state or if the electron density near the impurities is different from $\nu = 1/3$ in the initial state. The change in the angular momentum M of the initial level upon a change in the parameters of the systems leads to a sharp shift and a complete transformation of the emission spectrum. The arrangement of satellites and the intensity distribution in them can be used to draw conclusions about the lower part of the spectrum of elementary excitations in a nearly ideal system (a roton gap) or in a system perturbed by impurities. One cannot determine the size of the gap for the formation of free quasiparticle–hole pairs from this spectrum.

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