Stationary electrovacuum generalization of the Schwarzschild solution which is distinct from the Kerr–Newman metric

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An exact, asymptotically planar solution is derived in algebraic form for the Einstein–Maxwell equations. This solution describes the gravitational field of a stationary, axisymmetric, charged mass. This solution has the Schwarzschild metric as its static vacuum limit and is distinct from the familiar Kerr–Newman solution.

Castejon-Amenedo *et al.*[1] have derived an exact solution of Einstein's equations in vacuum which, like the well-known Kerr solution,[2] contains two physical parameters (mass and rotation) and which becomes the Schwarzschild metric in the case of a zero angular momentum. In the present paper we derive a charged generalization of that solution which contains the additional parameter α, which is a measure of the electric charge of the source and of the magnetic dipole moment μ which it generates. As in the case of the Kerr–Newman metric,[3] our solution allows a transition to a Reissner–Nordström metric[4] for the field of a static, spherically symmetric charged mass.

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This new metric, which is constructed by applying a Kramer–Neugebauer transformation[5] to the solution of Ref. 1, is determined by two complex Ernst potentials[6]

$$\epsilon = A_-/A_+; \qquad \Psi = -2\alpha B/A_+; \tag{1}$$

$$A_\mp = (p+1)(x-y)^4[(1-\alpha^2)x \mp (1+\alpha^2)]$$
$$\mp q^2(x^2-1)[(x\pm 1)^2(1\mp y) + \alpha^2(x\mp 1)^2(1\pm y)]$$
$$+iq\{(x-y)^4[(1-\alpha^2)x \pm (1+\alpha^2)]$$
$$\mp (p+1)(x^2-1)[(x\mp 1)^2(1\pm y) + \alpha^2(x\pm 1)^2(1\mp y)]\};$$
$$B = (p+1)(x-y)^4 + q^2(x^2-1)(x^2-2xy+1)$$
$$+iq[(p+1)(x^2-1)(x^2-2xy+1) - (x-y)^4],$$

where α, p, and q are real constants. The constants p and q are related by $p^2 - q^2 = 1$. From ϵ and Ψ we find the functions f, γ, and ω which fall in the Papapetrou metric interval[7]

$$ds^2 = k^2 f^{-1}\left[e^{2\gamma}(x^2-y^2)\left(\frac{dx^2}{x^2-1} + \frac{dy^2}{1-y^2}\right) + (x^2-1)(1-y^2)d\varphi^2\right] - f(dt - \omega d\varphi)^2 \tag{2}$$

(k is a constant). In the case at hand, the metric functions are (see Ref. 8, for example, for the corresponding equations for f, γ, and ω)

$$f = 2p(1-\alpha^2)^2(x^2-1)C/D;$$

$$e^{2\gamma} = \frac{x^2-1}{x^2-y^2}\frac{C}{(x-y)^8}; \qquad \omega = -\frac{2kq(1-y^2)E}{p(1-\alpha^2)^2 C}; \tag{3}$$

$$C = (x-y)^8 - q^2(x^2-1)^3(1-y^2);$$

$$D = (p+1)\{(x-y)^4[(1-\alpha^2)x+1+\alpha^2]$$
$$+(p-1)(x^2-1)[(x-1)^2(1+y)+\alpha^2(x+1)^2(1-y)]\}^2$$
$$+(p-1)\{(x-y)^4[(1-\alpha^2)x-1-\alpha^2]$$
$$+(p+1)(x^2-1)[(x+1)^2(1-y)+\alpha^2(x-1)^2(1+y)]\}^2;$$

$$E = (x-y)^5[(1-\alpha^4)(3x^2-3xy+y^2+1) + (1+\alpha^4)(3px-py)]$$
$$+q^2(x^2-1)^3[(1-\alpha^4)(x-2y) + p(1+\alpha^4)].$$

For the solution found, the total mass of the source (M), its angular momentum J, its charge Q, and its magnetic dipole moment μ can be found from (1) and (3) through the substitutions

$$x = (r - M)/k; \qquad y = \cos\theta \tag{4}$$

in the limit $r \to \infty$. The results are the expressions

$$M = \frac{kp(1+\alpha^2)}{1-\alpha^2}; \qquad J = \frac{k^2 q(1+\alpha^2)(3+q^2)}{p(1-\alpha^2)};$$
$$Q = -\frac{2\alpha kp}{1-\alpha^2}; \qquad \mu = -\frac{2\alpha k^2 q(p^2+1)}{p(1-\alpha^2)}. \tag{5}$$

It is a straightforward matter to verify that with $\alpha = 0$, $q \neq 0$ (in the absence of an electromagnetic field) solutions (1) and (3) becomes the metric of Ref. 1, while in the case $q = 0$, $\alpha \neq 0$ they become the Reissner–Nordström solution. In the case $\alpha = q = 0$, we obtain the Schwartzschild metric. Analysis of the scalar invariants I_1 and I_2 of the Weyl spinor[9] shows that the solution is generally of Petrov type I, degenerating to type D for the parameter values $q = 0$ and $q = \alpha = 0$ and also at the symmetry axis ($y = \pm 1$). The solution found here is thus distinct from the Kerr–Newman metric, because of the different multipole structure of the metric of Ref. 1 and of the Kerr solution.

We note in conclusion that solution (1), (3), like the metric of Ref. 1, has an event horizon defined by the hypersurface $x = 1$. In contrast with the Kerr–Newman metric, however, which has a regular horizon, the horizon of this solution has one singular point (the pole $y = 1$).

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