Quantum fluctuations annihilate an optical soliton

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(Submitted 3 December 1990)
Pis'ma Zh. Eksp. Teor. Fiz. **53**, No. 2, 73–76 (25 January 1991)

Quantum effects which accompany the propagation of a Schrödinger soliton in an intrinsically nonlinear optical fiber lead to a gradual destruction of the soliton. The reason for the destruction is identified.

It is generally believed that an ideal fundamental soliton which satisfies the nonlinear Schrödinger equation is established without loss in a nonlinear fiber over an infinite propagation distance. A related concept is "self-purification," i.e., the elimination of classical fluctuations from a soliton. For example, an initial noisy phase modulation of a soliton first converts into an amplitude modulation in the course of the nonlinear propagation of the soliton and then "descends" onto the wings, gradually leaving behind itself a "pure" formed soliton.[1,2] The quantum theory derived for the evolution of pulses in nonlinear optical fibers[3-7] predicts that the quantum uncertainty in the phase will increase and that there will be a dispersive spreading of solitons.[7] However, the approximations used in the derivation restrict the applicability of this conclusion to the initial stage of the nonlinear propagation. Does the self-purification property have a compensating effect on the growth of quantum fluctuations in the transition to the far zone, or does the destabilizing effect on the quantum uncertainty increase at all times and ultimately lead to a destruction of the soliton? Our purpose in this letter is to answer this question.

We describe the evolution of the quantum field in the fiber by the nonlinear Schrödinger equation[3-7]

$$i\partial\phi(t,x)/\partial t = -\partial^2\phi(t,x)/\partial x^2 + 2c\phi^+(t,x)\phi(t,x)\phi(t,x), \tag{1}$$

where the operators $\phi(t,x)$ and $\phi^+(t,x)$ annihilate and create a photon in the Heisenberg representation. These operators depend on the normalized time t, which is a measure of the distance traversed and which is reckoned from the beginning of the propagation; they also depend on the normalized coordinate x, which determines the distance from the crest of the pulse. The parameter c is the cubic-nonlinearity parameter.

The transformation of a state vector of the system, $|\psi\rangle$, obeys the equation

$$i\hbar d|\psi> /dt = H|\psi>, \quad H = \hbar[\int \phi_x^+(x)\phi(x)dx + c\int \phi^+(x)\phi^+(x)\phi(x)\phi(x)dx]. \tag{2}$$

Here we are using a Hamiltonian H and operators $\phi(x)$ and $\phi^+(x)$ in the Schrödinger representation. The solution of (2) is conveniently written as a superposition of Fok states $|n,p\rangle$ with a definite number (n) of photons and a momentum p (Ref. 7):

$$|\psi>=\sum_n a_n\int g_n(p)e^{-iE(n,p)t}|n,p>dp, \qquad (3)$$

here $|n,p\rangle=(n!)^{-1/2}\int_{-\infty}^{\infty}f_{np}(x_1,\ldots,x_n)\phi^+(x_1)\ldots\phi^+(x_n)dx_1\ldots dx_n|0\rangle$; $|0\rangle$ corresponds to the vacuum; $f_{np}(x_1,\ldots,x_n)=[(n-1)!|c|^{n-1}/2\pi]^{1/2}\exp[ip\Sigma_{j=1}^n x_j+c/2\times\Sigma_{1\leqslant i<j\leqslant n}|x_jx_i|]$ and the energy is $E(n,p)=np^2-c^2n(n^2-1)/1/12$. If the momentum is initially a set of coherent modes, then a_n and $g_n(p)$ obey Poisson and Gaussian distributions, respectively: $a_n=\alpha^n e^{-n_0/2}/\sqrt{n!}$, $g_n(p)=\pi^{-1/4}\Delta p^{-1/2}e^{-(p-p_0)^2/2\Delta p^2-inpx_0}$, where $n_0=|\alpha|^2$ is the average number of photons in the pulse.

We define the shape of its envelope, $\langle N(x)\rangle\equiv\langle\psi(\phi^+(x)\phi(x)|\psi(x)|\psi\rangle$:

$$<N(x)>\simeq\frac{2e^{-n_0}}{|c|}\sum_n\frac{n^2n_0^n}{n!}\int_{-\infty}^{\infty}p\,\mathrm{sh}^{-1}(2\pi p/|c|)\exp[-(\Delta p^{-2}$$

$$+4t^2n^2\Delta p^2)p^2+i2n(x-x_0-2p_0t)p]dp. \qquad (4)$$

The approximate equality in (4) concerns only the average number of photons. In other words, the condition $n_0\gg 1$ must hold; this condition is completely feasible in practice. No restrictions have been imposed on the range (the parameter t) or any other quantity in (4).

Analysis of this relation shows that the soliton spreads out continuously with increasing t. The characteristic spreading time depends on the relation between the nonlinearity parameter c and the width of the momentum distribution, Δp:

$$t_{\mathrm{char}}\simeq\begin{cases}2/n_0|c|\Delta p & \text{for}\quad \Delta p\gg|c|, & (5)\\ \sqrt{2}/n_0|c|\Delta p & \text{for}\quad \Delta p\simeq|c|, & (6)\\ \sqrt{3}/2n_0\Delta p^2 & \text{for}\quad \Delta p\ll|c|. & (7)\end{cases}$$

The validity of these estimates is confirmed by calculations whose results are reflected in Fig. 1. We see that the soliton suffers a complete degradation in the course of the nonlinear propagation.

To explain the results, we use the following model. The original soliton which enters the fiber as a set of modes in coherent states differing in amplitude can be described as a superposition of a classical envelope with a hyperbolic-secant shape (a regular component) and quantum fluctuations of the vacuum (a noise). The original noise modulation (which is present only within the length of the soliton) "drops" onto the wings, and the soliton undergoes a progressive "self-purification." In contrast, it cannot become totally free of the steady-state vacuum noise, since this "dropping" onto the wings is accompanied by an "influx" of fluctuations which were originally outside the soliton. These effects, which are working in opposite directions, are not balanced, and the picture does not stabilize. Let us see why. We consider the nonlinear evolution of a vacuum noise in the presence of an intense regular component of the soliton. We use the Heisenberg picture of nonlinear Schrödinger equation (1). We

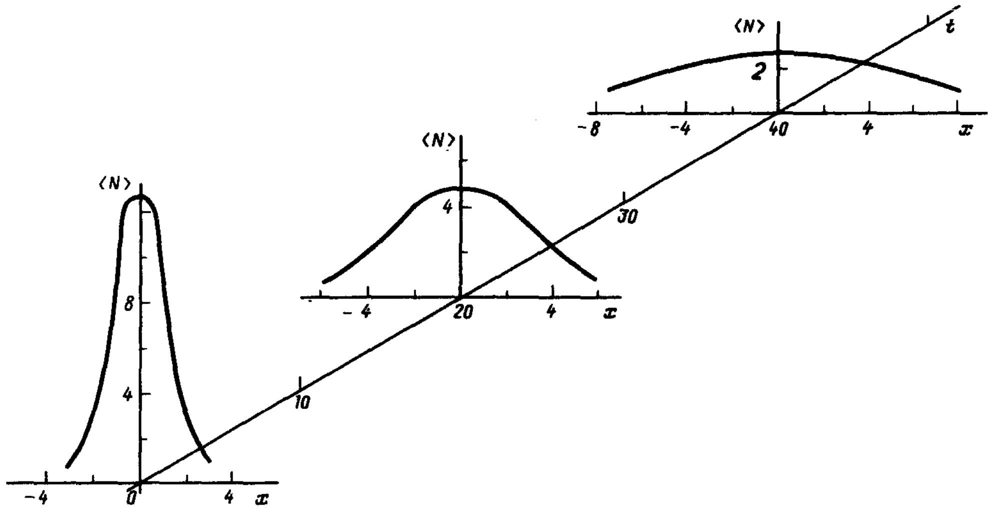

FIG. 1. Degradation of a soliton in the course of its nonlinear propagation ($|c| = \pi \cdot 10^{-2}$, $n_0 = 40$, $\Delta p = 0.1$, $p_0 = x_0 = 0$).

linearize this equation in terms of the fluctuating components, and for simplicity we analyze the single-mode interaction regime. We then find that the average number of noisy photons is $\langle N_n \rangle = \varphi^2$, where $\varphi = t(cn_0)^2/2$ is the nonlinear phase shift acquired in the course of the nonlinear propagation. Here n_0 is the average number of photons in a mode. There is accordingly a continuous increase in the noise intensity as a result of a pumping of photons from the regular component into the fluctuating component through a parametric four-wave mixing. The growth of the fluctuations is accompanied by a depletion of the soliton itself, which serves as a pump for the vacuum noise that is being amplified, since the total number of photons must remain constant if there is no loss. As a result of these irreversible processes, the soliton gradually spreads out, and it ultimately suffers total degradation.

Let us estimate the maximum possible range of a soliton in an optical fiber. According to (4), a fundamental soliton which can be described classically, i.e., whose envelope has a hyperbolic-secant shape, can exist in a fiber only under the condition $|c| < \Delta p$. Assuming $\Delta p \simeq |c|$ in the limiting case corresponding to a minimal spreading, we find from (6)

$$t_{\text{lim}} \simeq \sqrt{2}/n_0c^2 \equiv n_0T\sqrt{2}/8\pi \simeq n_0T/20. \tag{8}$$

Here the soliton period is $T = 8\pi/(n_0^2c^2)$; this is the time over which a nonlinear phase shift of 2π is built up. Since $n_0 \gg 1$ in real situations, t_{limb} is substantially greater than T. Quantum-mechanical effects will thus be manifested only at ultralong ranges or in media with a pronounced nonlinearity, i.e., under conditions such that other factors (a loss, a nonuniformity of the fiber, an inertia of the nonlinearity, etc.) will have a stronger destabilizing effect in practice. At present, solitons with a peak power of 10^{-2} W, a duration of 60 ps, and a wavelength of 1.56 μm can cover a distance of 10^4 km in a quartz fiber. In this case we would have $t \simeq 70$ T, $n_0 \simeq 4.8 \times 10^6$, and $t_{\rm limb}/t \simeq 3.4 \times 10^3$. If the effects discussed here are to be seen, it will thus be necessary to increase the path length or the nonlinearity by about two or three orders of magnitude. Whether such experiments can actually be carried out remains to be seen. However, the quantum degradation of a soliton imposes a fundamental upper limit on the propagation path of a soliton. This restriction is undoubtedly important.

I wish to thank V. A. Vysloukh for useful discussions.

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Translated by D. Parsons