Instability of a state of a quantum Hall effect in 2D spin systems

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(Submitted 25 January 1991)
Pis'ma Zh. Eksp. Teor. Fiz. **53**, No. 2, 87–89 (25 January 1991)

A special generalization of the nonrelativistic Goldstone theorem is used to prove the existence of gapless modes in a state (proposed previously) of a quantum Hall effect for 2D antiferromagnets. In the absence of holes, this state therefore could not be the ground state for any spin system with a short-range exchange.

Research on the high-T_c superconductors has attracted increased interest to disordered 2D magnetic systems. In several papers, the disruption of antiferromagnetic order (at absolute zero) has been linked with a spontaneous breaking of 2D parity and symmetry under time reversal. One particular scenario which has been proposed[1,2] is based on a mapping of a frustrated Heisenberg antiferromagnet into a system of impenetrable bosons in a state of a quantum Hall effect (QHE) with a half filling ($m = 2$). The conditions for the occurrence of a QHE state and of the related state of a chiral spin liquid[3] in 2D spin systems have not been clarified.[4] The importance of exact results in this connection has been stressed.[2]

In the present letter we show that a QHE state cannot be the ground state for any Heisenberg antiferromagnet with an exchange interaction of finite range. Through a special generalization of the nonrelativistic Goldstone theorem,[5] we prove that there are two gapless modes in this state. The presence of these modes is known to be incompatible with a stability of the QHE. We believe that Laughlin's exact QHE solution[2] for a model system is related to the long-range effect in that model.

Gapless modes in the state of Ref. 1 correspond to a spontaneous breaking of SU(2) symmetry under spin flip or, more precisely, to a part of this symmetry: two generators of the three, S^+ and S^-, while S^z remains a quantum number. This symmetry breaking occurs in the thermodynamic limit, despite the circumstance that the wave function which has been proposed has been a spin singlet for any finite number of atoms.[1,2] This symmetry breaking is associated with the circumstance that the QHE wave function is not a purely quantum state when there is a finite number of particles.[6] A situation of this sort is familiar from the theory of antiferromagnetism: A Heisenberg antiferromagnet with an interaction exclusively between nearest neighbors on a square or cubic lattice satisfies a corresponding singlet sum rule.[7] Nevertheless, there is a long-range order in the thermodynamic limit. The contradiction can be resolved[8] by noting that the direction of the magnetization can be chosen by introducing a small anisotropy, which is then allowed to go to zero after the thermodynamic limit is taken. Below, an expectation value over a state always means a quasi-expectation value of this sort.[9]

Working from the effective action for elementary excitations, one can reach the

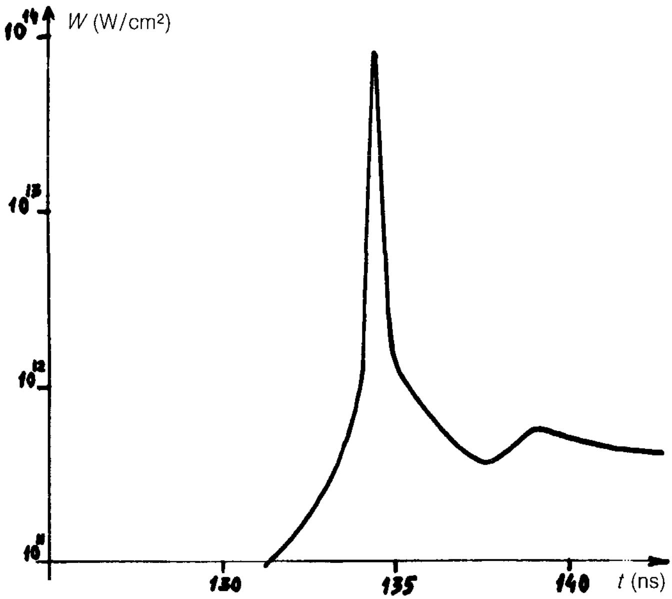

FIG. 2. Radiation pulse from a unit surface area of the xenon liner.

With increasing amplitude of the current flowing through the liner, the optimum liner mass and the density increase in proportion to the square of the current amplitude.[7] As a result, the strength of the radiation pulse and the pressure, both of which are proportional to the liner density according to (2), increase. The use of liners thus makes it possible to carry out studies of, for example, equations of state in the multimegabar pressure range.

We wish to thank A. Yu. Krukovskiĭ and K. V. Skorovarov for collaboration in the calculations.

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Translated by D. Parsons

conclusion that there is a breaking of SU(2) in a QHE state.[10] We show below that this symmetry breaking is spontaneous, and we find the corresponding order parameter.

We consider a Heisenberg antiferromagnet on an arbitrary 2D lattice: $H=\Sigma_{ij}J_{ij}\vec{S}_i\vec{S}_j$, where the exchange interaction has a finite radius ($J_{ij}=0$ for $|i-j|>\rho$). The spin components are expressed in terms of the operators of impenetrable bosons:[1] $S_i^- = c_i$, $S_i^+ = c_i^+$, $S_i^z = c_i^+ c_i - 1/2$. This replacement is exact (in the spin-1/2 representation) if we assume that the boson operators satisfy Bose relations for different nodes but satisfy Fermi relations for a common node (the nonvanishing anticommutator is $\{c_1^+, c_i\} = 1$). A mixed algebra of this sort is incompatible with an adiabatic transition[1] to the continuous limit. On this basis we would expect that there would be no gap in the spectrum.

The elementary excitations of the QHE state are vortices with a flux of plus and minus one, called a "quasiparticle" and a "quasihole," respectively.[1] They can be localized at any point in the plane, not necessarily at a node. Since the total-spin operators are defined by $\vec{S}=\Sigma_{i\in V}\vec{S}_i$, where $V\to\infty$ after the thermodynamic limit is taken, these excitations have $S^z = \pm 1/2$. If the SU(2) symmetry were not broken, the ground state in the thermodynamic limit would be a spin singlet, and the quasiparticle and quasihole would form a spin doublet. We can show that these conditions cannot be satisfied simultaneously. We consider the order parameter

$$< F|\psi_{z_0}^+\psi_{z_0}^- + \psi_{z_0}^-\psi_{z_0}^+|F> \neq 0, \quad (1)$$

where $|F\rangle$ is the state of the quantum Hall effect; the operators ψ^+, ψ^- create a quasiparticle and a quasihole, respectively; and z_0 is not a node of the lattice. If ψ^+, ψ^- form a doublet, the left side of (1) is the expectation value of the $S^z=0$ component of the triplet, so that (1) does indeed imply a spontaneous breaking of SU(2). To see that the order parameter is nonzero, we use the variational expressions[1] $\psi^+ = \Pi_i(z_i - z_0)$, $\psi^- = \Pi_j(\eta_j - z_0)$, where the product in the first case is over nodes occupied by bosons, while that in the second is over vacant nodes. We obviously have $\psi^+\psi^- + \psi^-\psi^+ = 2\Pi_\zeta(\zeta - z_0)$; this is a c-number function, since the product is now over all the lattice nodes. This function is nonzero if z_0 is not a lattice node. We have thus proved (1).

Since order parameter (1) contains nonlocal quasiparticle operators, the conclusion that gapless modes exist requires some generalization of the nonrelativistic Goldstone theorem. The assertion that there is a spontaneous symmetry breaking is equivalent to $\langle F|\Sigma_{i\in V}S_i^- A|F\rangle \neq 0$, where $V\to\infty$, and A is a nonlocal operator with $S^z = +1$ (e.g., $\psi^+\psi^+$). We consider the correlation function

$$G(t) =< F|[\Sigma_{i\in U}S_i^-(t),\ \Sigma_{j\in V}S_j^z(0)]A(0)|F>, \quad (2)$$

where $U\supset V\to\infty$. In this limit, the time dependence of (2) is determined by the energies of intermediate states in the commutator, since S^z generates zero when acting on $|F\rangle$. On the other hand, the commutator contains only short-range variables, so we can now employ the known arguments[5] without changes. We have thus shown that $\Sigma_{i\in V}S_i^-$ and, correspondingly, $\Sigma_{i\in V}S_i^+$ generate two types of gapless excitations reminiscent of antiferromagnetic magnons when they operate on $|F\rangle$. Our results indicate that the state of the quantum Hall effect of 2D electron systems is unstable in the absence of holes.

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