Anomalous temperature dependence of the first critical field of a ceramic high-T_c superconductor

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Mean-field equations are constructed to describe a model of a granular superconductor at nonzero temperatures. The temperature dependence of the first critical field, $H_{c1}(T)$, is analyzed. It should have an inflection point.

If the links between grains in a ceramic are sufficiently weak, a Josephson medium in a high-T_c superconductor in a weak magnetic field can be described by the XY model (Ref. 1, for example). In general, the approximation used in Ref. 1 is valid only at low temperatures. It cannot be used for a self-consistent calculation of the critical temperature (T_{cJ}) for the establishment of a coherent state or to study the temperature dependence of the penetration depth, of the critical fields, etc. The model constructed below is valid at all temperatures ($0 < T < T_{cJ}$).

Under the assumption that the grains are centered at the sites of a cubic lattice (this assumption has no effect on the results in the continuum limit), we have the energy functional[2]

$$H[\theta] = \sum_{xx'} \left\{ -E_J \cos[\theta_x - \theta_{x'} - \frac{2\pi}{\Phi_0} L\vec{A}_{xx'}\vec{e}_{xx'}] + \frac{L^3}{8\pi\mu}\vec{B}^2_{xx'} \right\}. \quad (1)$$

Here θ_x is the value of the phase in the grain centered at x, the sum is over pairs of nearest neighbors, $E_J = L^2 j_c \hbar/2e$ is the energy of an isolated junction, j_c is the critical current density of the junction, L is the grain size, $\vec{e}_{xx'}$ is a unit vector directed from site x to site x', $\vec{A}_{xx'}$ is the vector potential at a junction between grains, $\vec{B}_{xx'}$ is the magnetic induction, μ is the magnetic permeability of the Josephson medium, and Φ_0 is the magnetic flux quantum. In the case $L \gg \lambda$ (where λ is the London penetration depth of a grain) we have $\mu \cong \lambda/L$. In the continuum limit we have $\vec{B} = \mu\vec{H}_J = \mathrm{curl}\,\vec{A}$, where H_J is the field at a junction.

We should point out immediately that in this letter we are not concerned with spin-glass properties which arise when there is a disorder in the system (when there is a randomness in the values of E_J).

The macroscopic properties of a granular superconductor are determined by the free energy F of the statistical system with Hamiltonian (1). The corresponding partition function is

$$Z = \exp\left(-\frac{F}{T}\right) = \int \Pi_x \frac{d\theta_x}{2\pi} \exp\left(-\frac{H[\theta]}{T}\right). \quad (2)$$

For an analysis we use the standard procedure,[3,4] which makes it possible to construct mean-field equations for the order parameter. In the case at hand, the order parameter is the quantity $\exp[i\theta(x)]$, averaged over the ensemble.[5] In the continuum limit we have

$$H[\theta] = -3E_J \int \frac{d^3x}{L^3} \phi^*[\theta] \left(1 + \frac{L^2}{6}\hat{D}^2\right) \phi[\theta] + \frac{1}{8\pi\mu} \int d^3x \vec{B}^2, \tag{3}$$

where $\phi[\theta] = \exp(i\theta)$, and $\hat{D}$ means the "long" derivative ($\hat{D}_i = \partial/\partial x_i - 2\pi i A_i/\Phi_0$). Using the Hubbard–Stratonovich identity,[3,4] in partition function (2) with Hamiltonian (3), we can switch from an integration over the field θ to an integration over the new field ψ, whose equilibrium value is directly related to the order parameter [see (7)]. We finally find the following expressions for the partition function and the effective Hamiltonian in terms of the field ψ:

$$Z = \int \Pi_x d\psi^* d\psi \exp\left\{-\frac{1}{T}\int d^3x \left(H_{eff}[\psi] + \frac{1}{8\pi\mu}\vec{B}^2\right)\right\}, \tag{4}$$

$$H_{eff} = z\left|\left(\nabla - \frac{2\pi i}{\Phi_0}\vec{A}\right)\psi\right|^2 + a|\psi|^2 - \frac{T}{L^3}\ln(I_0(|\psi|)), \tag{5}$$

where $z = T^2/(72E_J L)$, $a = T^2/(12E_J L^3)$, and $I_0(s)$ is the modified Bessel function. For $|\psi| \ll 1$ (this condition holds near T_{cJ}), the nonlinear term in (5) can be expanded in a series, and we obtain the standard Ginzburg–Landau model. A mean-field equation is found by varying (5):

$$z(i\nabla + \frac{2\pi}{\Phi_0}\vec{A})^2\psi + a\psi - \frac{T}{2L^3}\frac{I_1(|\psi|)\psi}{I_0(|\psi|)|\psi|} = 0. \tag{6}$$

In the mean-field approximation the order parameter $\langle\psi\rangle$ is $\langle\psi\rangle = I_1(|\psi|)/I_0(|\psi|)$ ($\langle\ \rangle$ means a thermodynamic average). In the absence of an external field, the equilibrium value of ψ_0 is found from the equation

$$\frac{T|\psi|}{6E_J} = \frac{I_1(|\psi|)}{I_0(|\psi|)} = <\phi>, \tag{7}$$

which has a nontrivial solution only at $T < T_{cJ}$. The value of T_{cJ} is found from the equation $3E_J(T) = T$. Near T_{cJ} we have $|\psi_0|^2 = 8(1 - T/T_{cJ})$, and at small T we have $|\psi_0| = 6E_J(0)/T$. From (5) we find expressions for the penetration depth δ, the coherence length ξ, and the first critical field H_{c1}:

$$\frac{1}{\delta^2} = \frac{4\pi^3\mu T^2}{9\Phi_0^2 E_J(T)L}|\psi_0|^2; \quad \frac{1}{\xi^2} = \frac{12}{L^2}\left(1 - \frac{3E_J(T)}{T} + \frac{|\psi_0|^2}{4}\frac{T}{3E_J(T)}\right), \tag{8}$$

$$H_{c1} = \frac{\Phi_0}{4\pi\delta^2\mu}\ln\frac{\delta}{\xi}. \tag{9}$$

The physical meaning of the upper critical field for a Josephson medium, $H_{c2} = \Phi_0(2\pi\xi^2\mu)$, is discussed in Ref. 1.

Let us use these expressions to analyze the behavior of the function $H_{c1}(T)$. At

$E_J = \text{const}$, the function $H_{c1}(T)$ has the customary form for a homogeneous supercondutor. The situation changes when the temperature dependence of E_J is taken into account. An inflection point appears on the $H_{c1}(T)$ curve. This result can be explained at a qualitative level as follows. If we ignore the temperature dependence of the quantity in the logarithm in (9), we find the following asymptotic behavior of $H_{c1}(T)$ at temperatures close to T_{cJ}:

$$H_{c1}(T) = 2H_{c1}(0)\frac{E_J(T_{cJ})}{E_J(0)}(1 - T/T_{cJ}). \tag{10}$$

Since the relation

$$E_J(T_{cJ}) \ll E_J(0), \tag{11}$$

usually holds, we immediately see that the linear asymptotic behavior in (10) cannot join smoothly with the value of the critical field in the plateau, $H_{c1}(T) \cong H_{c1}(0)$, at

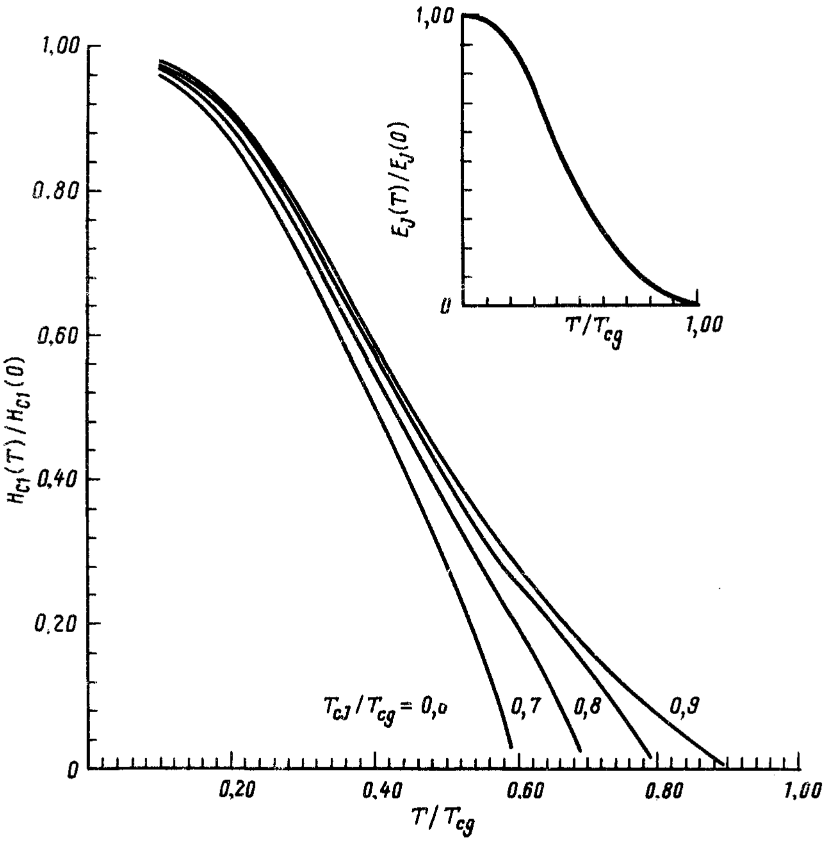

FIG. 1. Temperature dependence of the first critical field of a granular superconductor for various values of T_{cJ}. Inset: Experimental temperature dependence $E_J(T)$.

low temperatures in the absence of an inflection point. In the case of an *S–I–S* junction, this feature is essentially unseen. The reason is that condition (11) holds only in a very small neighborhood of T_{cg}, where T_{cg} is the superconducting transition temperature of the grains. However, E_J for an *S–I–S* junction increases rapidly with decreasing temperature, the linear asymptotic expression in (10) becomes inapplicable almost immediately, and the inflection point is essentially unseen. The anomalous behavior of $H_{c1}(T)$ can be seen clearly if, under condition (11), the function $E_J(T)$ varies sufficiently slowly near T_{cJ}. This behavior is characteristic of *S–N–S* junctions[6] and also of *S–I–S* junctions with a short coherence length.[7] The quantity $E_J(T)$ which is realized in ceramic high-T_c superconductors[8,9] also satisfies this condition. The inset in Fig. 1 shows the experimental temperature dependence $E_J(T)$ for the single-phase ceramic metal oxide $YBa_2Cu_3O_x$ (the curve is an approximation of the experimental data shown in Fig. 5 of Ref. 8). Curves of $H_{c1}(T)$ calculated from this $E_J(T)$ dependence for various values of T_{cJ} are shown in Fig. 1. We see that the function $H_{c1}(T)$ is of an anomalous two-step nature. It is difficult to find $H_{c1}(T)$ experimentally because of the low values of the first critical field. We are aware of only a single measurement[10] of the temperature dependence of H_{c1}. The experimental curve of $H_{c1}(T)$ shown in Fig. 5 in Ref. 10 has a typical inflection point.

An anomaly of this sort on the temperature dependence $H_{c1}(T)$ is usually linked with the existence of a second superconducting phase, with a lower transition temperature. In our model, the anomaly is present in the case of a single-phase granular superconductor.

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