Method for cooling relativistic electron beams

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A method for cooling relativistic electron beams is proposed. This method is based on a nonlinear mechanism for the interaction of the beam with an electromagnetic wave in an undulator field combined with a constant, uniform magnetic field.

The quality of an electron beam (i.e., the value—preferably small—of the energy spread $\delta\gamma/\gamma$, where $\gamma = E/mc^2$, and E is the beam energy) is of decisive importance for realizing a free-electron x-ray laser, since the gain per pass is proportional to N^3 (N is the number of periods of the undulator) only if $\delta\gamma/\gamma < 1/2N$. The absence of an energy spread is of fundamental importance, since the angular divergence of a beam can be eliminated by magnetic-optics devices.

We would like to propose a method for reducing the energy spread of an electron beam (i.e., for cooling the beam). This method is based on an inelastic interaction of the electrons with an intense electromagnetic wave in the varying field of an undulator and in a static magnetic field directed parallel to the magnetic component of the field of the laser wave (this layout was proposed in Ref. 1 for raising the efficiency of free-electron lasers). The cooling of the beam results from an "autoprocess": When the amplitudes of the wave field, of the undulator field, and of the static magnetic field satisfy certain relations, all the electrons acquire an energy increase (exclusively an increase), and the increase becomes larger in size with decreasing initial energy of the electrons. As a result, there is the possibility in principle of cooling an electron beam by several orders of magnitude.

Let us assume that an electromagnetic wave with a frequency ω, a wave vector k_s, and a field amplitude E_s is propagating along the z axis in an undulator with a period $\lambda_w = 2\pi/k_w$ and a field amplitude B_w. We assume that there is an additional magnetic field B_0, directed parallel to B_s and B_w.

A resonant exchange of energy between an electron and the electromagnetic wave occurs if the longitudinal velocity of the electron, v_z, is equal to the phase velocity $v_{ph} = \omega/k$, where $k = k_s + k_w$, of the combinational (ponderomotive) wave $a_s a_w \cos\psi$, where $\psi = kz - \omega t$, $a_s = eE_s/m\omega c^2$, and $a_\omega = eB_\omega/k_\omega mc^2$. The interaction of an electron with the combinational wave is conveniently described in Hamiltonian form[2,3] as the motion of a particle in the ponderomotive potential $U(\psi)$:

$$U(\psi) = -\frac{k_s}{2k_w\gamma} a_s a_w (\cos\psi + \alpha\psi), \tag{1}$$

$$\alpha = \frac{\sqrt{2}\Omega\mu}{ck_w a_s a_w} \left(\frac{\Delta}{\gamma_{cr}}\right)^{1/2}. \tag{2}$$

Here $\Omega = eB_0/mc$, $\mu^2 = 1 + a_\omega^2/2$, $\Delta = \gamma - \gamma_{cr}$, and $\gamma_{cr} = k_s\mu^2/2k_\omega$. The nature of the interaction of the electrons with the wave is determined by the shape of the ponderomotive potential $U(\psi)$. If $\alpha > 1$, the interaction is linear in the power of the laser wave, and the efficiency of the interaction is low.[3] There is no well in the ponderomotive potential in this case. If $a < 1$, there can be a trapping in the well of the ponderomotive potential, and there may also be a reflection regime (a single pass of the particle through the resonance condition).[3] Effective cooling can occur only in the second case, since an energy spread on the order of the depth of the well is always retained when particles are captured in a well.

Let us examine the cooling regime in more detail. The electron is not captured by the well of the ponderomotive potential if the deviation of the electron's energy from the resonant value is greater than the depth of the well:

$$\Delta > \Delta_R = \gamma_{cr}^{1/2}|U(\psi_R) - U(\pi - \psi_R)|^{1/2}, \tag{3}$$

where $\psi_R = \arcsin\alpha$ and $\alpha < 1$. The change in the energy of the electron (the efficiency η) is proportional to the area of the well,[2,3] $S(\alpha)$:

$$\eta = 4(2a_s a_w)^{1/2}S(\alpha)/\pi\mu. \tag{4}$$

The function $S(\alpha)$ decreases from one to zero as α increases from zero to one.[2] It is not difficult to see from (2) and (4) that the efficiency falls off with increasing value of the parameter α, which is in turn an analytic and monotonic function of Δ if the initial conditions are far from resonance, $\Delta > \Delta_R$ (the electrons are not captured in the well). If there is a small spread in terms of the initial energy of the electrons, $\delta\gamma_0 < \Delta$, the change in energy as a result of the interaction is thus proportional to the initial energy spread. If the initial conditions are subsequently chosen in such a way that the phase intervals $\Delta\psi_f$ forbidden for reflection correspond to values $\sin\Delta\psi_f > 0$ (the electron acceleration regime), the proportionality factor will be positive. With increasing initial energy of the electron, the electron acquires less energy as a result of the interaction.

Figure 1 shows the ponderomotive potential in (1). Also shown here is a rough sketch of the energy acquired by the electron as a function of its initial energy. It follows from Fig. 1 that for an initial energy spread $\delta\gamma_0/\gamma_{cr} \ll (a_s a_\omega)^{1/2}/\mu$ it is possible to control the final spread. A *cooling* of the beam is possible if the energy spread acquired by the electron in the course of the interaction, $\delta U = (a_s a_\omega \alpha)^{1/2}/\mu$, which corresponds to a phase interval $\Delta\psi_a$ which is allowed for cooling (Fig. 1), is much smaller than the initial spread: $\delta U \ll \delta\gamma_0$.

We can now move on to formulate conditions under which an initial spread $\delta\gamma_0$ is erased [to within terms $(\delta\gamma_0)^2$]. Let us assume that the deviation of the center of the energy distribution of the beam from the resonance condition is considerably greater than the energy spread: $\delta\gamma_0 \ll \Delta_0$. We expand η in a series in the parameter $\delta\gamma_0/\Delta_0$ and retain terms up to $(\delta\gamma_0/\Delta_0)^3$:

$$\eta = \eta_0 + \eta'_\Delta(\delta\gamma_0/\Delta_0) - \eta''_{\Delta\Delta}(\delta\gamma_0/\Delta_0)^2, \tag{5}$$

where $\eta'_\Delta = [2(2a_s a_\omega)^{1/2}/\pi\mu]\alpha S'_\alpha(\alpha)$ and $\eta''_{\Delta\Delta} = [(2a_s a_\omega/2)^{1/2}/\pi\mu] \times [\alpha S'_\alpha(\alpha) - \alpha^2 S''_{\alpha\alpha}(\alpha)]$.

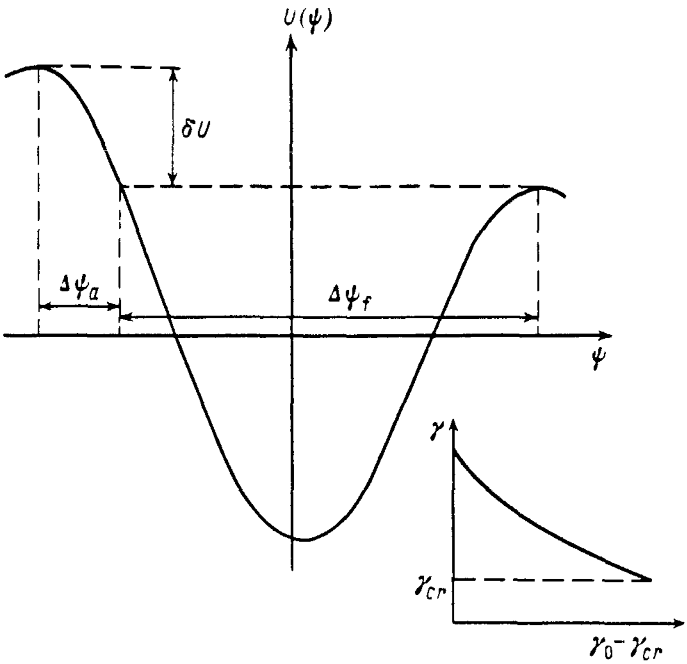

FIG. 1. The ponderomotive potential $U(\psi)$. The inset shows the final energy of the electron as a function of its initial energy.

It follows from Refs. 2 and 3 that we have $S''_{\alpha\alpha}(\alpha) \ll S'_{\alpha}(\alpha)$. Ignoring this small quantity, we can write the final energy spread $\delta\gamma$ (i.e., the spread after the interaction) as a function of the initial spread:

$$\delta\gamma = \delta\gamma_0 \left(1 - \frac{\gamma_{cr}\eta'_\Delta}{\Delta_0}\right) + \frac{\gamma_{cr}\eta'_\Delta}{2}\left(\frac{\delta\gamma_0}{\Delta_0}\right)^2. \tag{6}$$

Under the condition

$$\frac{\Delta_0}{\gamma_{cr}} = \frac{16S'^2(\alpha)}{\pi^2 a_s a_w}\left(\frac{\Omega}{ck_w}\right)^2, \tag{7}$$

the final energy spread is then

$$\delta\gamma/\gamma_{cr} = (\delta\gamma_0)^2/2\Delta_0\gamma_{cr} \tag{8}$$

$(\delta\gamma_0 \ll \Delta_0)$. In other words, the energy spread has been greatly reduced. The quantity Δ_0 must satisfy the inequality

$$\frac{(a_s a_w)^{1/2}}{\mu} S(\alpha) < \frac{\Delta_0}{\gamma_{cr}} < \frac{(a_s a_w)^2}{2\mu^2}\left(\frac{ck_w}{\Omega}\right)^2, \tag{9}$$

where the left side guarantees that the particles are reflected (not captured) [see (3)],

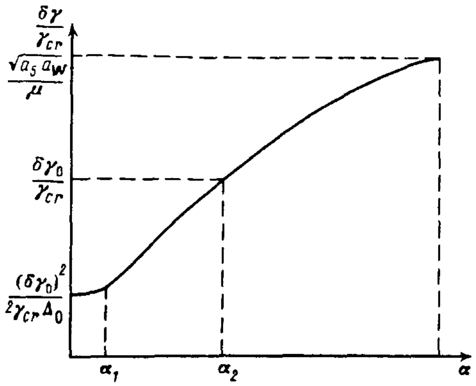

FIG. 2. Final energy spread of the electron as a function of the parameter α—$\alpha_1 = [(\delta\gamma_0)^4/(2\gamma_{cr}\Delta_0)^2]\,(\mu^2/a_s a_w)$ and $\alpha_2 = [\delta\gamma_0/\gamma_{cr}]^2\,(\mu^2/a_s a_w)$.

and the right side is the condition for the existence of forbidden phase intervals.[1] Conditions (7) and (9) determine the parameters which the apparatus must have. These conditions can clearly be satisfied simultaneously if

$$a_s a_w N^2/\mu^2 > 1. \tag{10}$$

In this case the quantity Δ_0 is close to $\gamma_{cr}(a_s a_w)^{1/2}/\mu$. It then follows from (8) that with $a_w^{1/2}/\mu \sim 1$ and the value $a_s \sim 10^{-2}$, which is feasible in practice, an initial spread $\delta\gamma_0/\gamma = 10^{-2}$–$10^{-3}$ can be reduced by two or three orders of magnitude.

The limiting value of the energy spread given by (8) is valid, however, if $\delta U/\gamma_{cr}$ is less than this value (Fig. 1). In the opposite case, the final spread will be limited to a value $\delta U/\gamma_{cr}$. If, on the other hand, the initial spread satisfies $\delta\gamma_0 < \delta U$, then the final spread may even be greater than the initial spread. Figure 2 shows a rough sketch of the final energy spread as a function of the parameter α under conditions (7) and (10). We see that in the region $(\delta\gamma_0/\gamma_{cr})^2\mu/a_s a_w$ there is an effective cooling of the electron beam. In this region, δU is essentially independent of the change in the electron's energy in the course of the interaction, because of the small value of the parameter $\Delta\psi_a/\Delta\psi_f$.

[1] V. A. Bazylev and A. V. Tulupov, Zh. Tekh. Fiz. **57**, 2221 (1987) [Sov. Phys. Tech. Phys. **32**, 1341 (1987)].

[2] N. Kroll, P. Morton, and M. N. Rosenbluth, in *Free-Electron Sources of Coherent Radiation* (ed. A. A. Rukhadze) [Russian transl.], Mir, Moscow, 1983, p. 69.

[3] V. A. Bazylev and A. V. Tulupov, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **32**, 1238 (1989).

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