Closed Abrikosov vortices in type-II superconductors

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(Submitted 8 January 1991)
Pis'ma Zh. Eksp. Teor. Fiz. **53**, No. 3, 150–153 (10 February 1991)

The magnetic field distribution, the magnetic flux, and the free energy of a closed toroidal vortex in a type-II superconductor are calculated in the London approximation. The thermodynamic condition for the excitation of such a vortex by a moving charge contains the fine-structure constant as a determining parameter.

The shape of a vortex region in a type-II superconductor generally reproduces the structure of the field lines of the external magnetic field $\vec{H}_0$. In the case of a uniform field $\vec{H}_0$, for example, there is a mixed state consisting of a two-dimensional lattice of rectilinear vortex regions.[1,2] More-complex vortex structures are possible; one such case is that in which a magnetic field is produced by a current flowing through the superconductor, and the vortex region envelops current lines.[3,4] In the present letter we analyze the properties of a closed Abrikosov vortex with a vortex region of toroidal structure.

Working in the London approximation, we calculate the magnetic field distribution in a closed toroidal vortex in an unbounded superconductor with a vortex region consisting of a circle of radius R_s (Fig. 1). In the cylindrical coordinate system (r, φ, z), whose $z=0$ plane coincides with the plane of the vortex region, the magnetic field has only an azimuthal component H, and the initial London equation[5] can be written in the form

$$\frac{\partial^2 H}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H}{\partial \rho} + \frac{\partial^2 H}{\partial \zeta^2} - \left(1 + \frac{1}{\rho^2}\right) H = -\frac{\Phi_0}{\lambda^2}\delta(\rho - \rho_s)\delta(\zeta). \qquad (1)$$

Here $\rho = r/\lambda$, $\zeta = z/\lambda$, $\rho_s = R_s/\lambda$, λ is the London penetration depth, and $\Phi_0 = \pi\hbar c/e$ is the flux quantum. Using Fourier-Bessel transforms, we find the following distribution for the dimensionless magnetic field $h(\rho,\zeta) = H\lambda^2/\Phi_0$ in the toroidal vortex:

$$h(\rho,\zeta) = \frac{\rho_s}{2}\int_0^\infty dq \frac{qJ_1(\rho q)J_1(\rho_s q)\exp(-|\zeta|\sqrt{1+q^2})}{\sqrt{1+q^2}}, \qquad (2)$$

where J_1 is the Bessel function of the first kind. Expression (2) has a logarithmic divergence at the center of the vortex region ($\rho = \rho_s$, $\zeta = 0$). The reason for this divergence is that London equation (1) is not valid in the region of the normal vortex core, with $|\rho - \rho_s| \leqslant \xi$, where ξ is the coherence length. The magnetic field in the calculations was thus limited (in the usual way) at distances ξ from the center of the vortex region.[5] The structure of a toroidal vortex with a large radius $R_s \gg \lambda$ is similar to that of a linear vortex, and the magnetic field in it varies significantly over distances

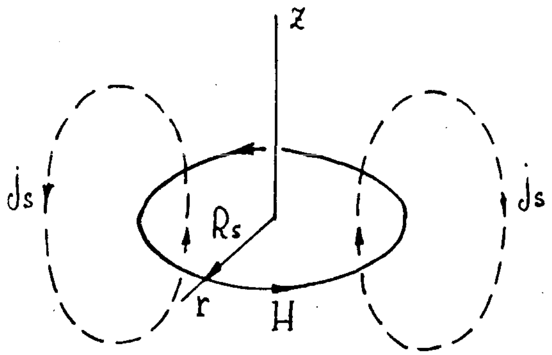

FIG. 1. Magnetic field H and current density $\vec{j}_s$ in a closed toroidal Abrikosov vortex.

on the order of λ. In the opposite limit $R_s \ll \lambda$, the spatial region occupied by the toroidal vortex is determined primarily by the radius R_s.

The free energy of the vortex in the London model can be expressed in terms of the magnetic field h_s at the center of the vortex region:[5]

$$C_v = \frac{\Phi_0^2}{8\pi\lambda^2} L_s h_s, \tag{3}$$

where $h_s = h(\rho = \rho_s, \zeta = 0)$, and $L_s = 2\pi R_s$ is the length of the vortex region. Figure 2 shows the field h_s and the energy C_v versus the radius R_s. Since the energy C_v increases monotonically with increasing R_s, a toroidal vortex is unstable, tending to contract toward the z axis and ultimately collapsing. This contraction of the vortex

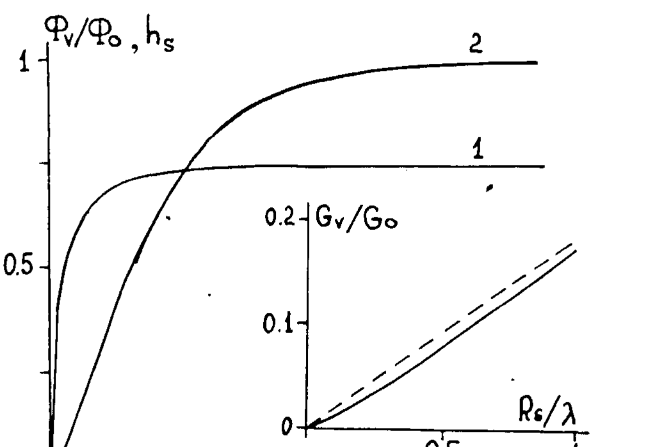

FIG. 2. 1—Magnetic field at the center of the vortex region, h_s; 2—the magnetic flux Φ_v; inset—the free energy G_v, all as functions of the radius of the vortex region, R_s. Shown for comparison by the dashed line is the free energy of a linear Abrikosov vortex with a length $L_s = 2\pi R_s$ ($\kappa = \lambda/\xi = 10^2$).

region is accompanied by jumps in the phase of the order parameter and by a local disruption of superconductivity. Calculations carried out on the stabilization of a toroidal vortex by a current flowing through a bounded cylinder show that a minimum appears on the curve of the free energy $G_v(\rho_s)$ at a certain value of the total current, and $G_v(\rho_s)$ goes negative as the current is increased further. The meaning here is that by using suitable current pulses one could create, fix, and annihilate closed toroidal vortices in a cylinder with an external current which stabilizes the vortices.

Integrating distribution (2) over the (ρ, ζ) half-plane, we can calculate the flux Φ_v in a toroidal vortex:

$$\Phi_v = \Phi_0 \rho_s \int_0^\infty dq \frac{J_1(\rho_s q)}{1+q^2}. \tag{4}$$

The flux Φ_v thus depends on the radius of the vortex region, R_s, and it reaches its asymptotic value Φ_0 at $R_s \gg \lambda$ (Fig. 2). Using an expression[5] for the superconducting current $\vec{j}_s$, we can write the following quantization condition:

$$\Phi_v + \frac{2\pi\lambda^3}{c} \int_{-\infty}^{\infty} d\varsigma j_{sz}(\rho = 0, \varsigma) = \Phi_0. \tag{5}$$

The contraction of a closed vortex is thus accompanied by an increase in the density of the superconducting current at the z axis.

Let us examine a possibility for exciting closed toroidal vortices in superconductors. The azimuthal structure which is required for forming a vortex is offered by the self-magnetic field H_q of a charge q which is moving at a velocity $V = \beta c$, where c is the velocity of light (Fig. 3).[6] Instead of dealing with the rigorous time-varying problem, we use the following thermodynamic condition to evaluate the possibility of

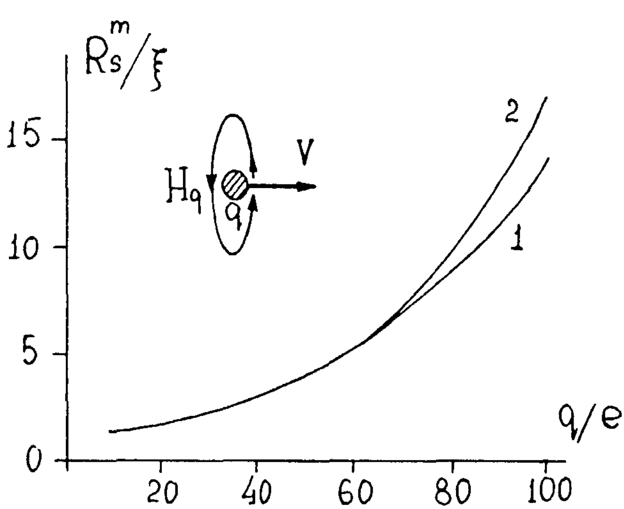

FIG. 3. Maximum permissible radius of a vortex region, $R_s^m = \lambda\rho_s^m$, versus the magnitude of the charge of the moving particle, q. 1—$\kappa = \lambda/\xi = 10^2$; 2—$\kappa = \lambda/\xi = 10^3$.

exciting a toroidal vortex:

$$C_v - \frac{1}{4\pi}\int d^3r(H_qH) \leq 0. \tag{6}$$

Inequality (6) imposes an upper limit $R_s^m(R_s \leqslant R_s^m)$ on the radius of a vortex region which can be excited in this fashion. For a charge in relativistic motion ($\beta \simeq 1$), $\rho_s^m = R_s^m/\lambda$ satisfies the equation

$$\frac{2}{\pi h(\rho_s^m, 0}\frac{\exp(\rho_s^m) - 1}{\rho_s^m} = \frac{\alpha^{-1}}{Z}. \tag{7}$$

Here $\alpha = e^2\hbar c$ is the fine-structure constant (spin-orbit constant), $Z = q/e$, and e is the charge of an electron. The only physical constant which determines the excitation of a toroidal vortex—which is essentially a macroscopic entity—is thus the fine-structure constant α. Figure 3 shows the maximum permissible vortex radius R_s^m versus the magnitude of the charge q. A formation of toroidal vortices might be seen, for example, in the characteristic energy loss of charged particles.

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Translated by D. Parsons