Anomalous magnetic moment and spontaneous symmetry breaking in scalar 3D electrodynamics

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(Submitted 4 January 1991)
Pis'ma Zh. Eksp. Teor. Fiz. **53**, No. 4, 177–179 (25 February 1991)

The consequences of spontaneous symmetry breaking are analyzed in an effective theory which describes the interaction between a charged scalar field an anomalous magnetic moment and a Chern–Simons–Maxwell Abelian field.

Effects associated with gauge interactions of particles having an anomalous magnetic moment have attracted research interest since the beginnings of quantum field theory, because of possible physical applications.

An interesting aspect of 3D space-time is that scalar particles "living" in it may have an anomalous magnetic moment.

In this letter we examine the effects of spontaneous symmetry breaking in a model which describes the gauge interaction of a charged scalar field with an anomalous magnetic moment. We will see that the spontaneous breaking of $U(I)$ symmetry may result in the generation of a Chern–Simons term (if there is no such term in the original action of the theory), while at the same time the Maxwell term may disappear as a result of cancelling contributions which arise in the course of the spontaneous breaking.

Let us recall the definition of the anomalous magnetic moment of a particle and discuss features of this moment in a $D=2+1$ space-time.

We begin with the Dirac equation for a charge spinor field of mass m with an anomalous magnetic moment which is interacting with an Abelian gauge field $A_m(x)$:

$$[(\partial_m + iA_m)\gamma^m - \frac{l}{2}F_{mn}\gamma^m\gamma^n]\Psi(x) = im\Psi(x), \tag{1}$$

where $\gamma^m - D = 2+1$ are the Dirac matrices [$m = 0,1,2$; the signature of the metric has been chose to be $(+,-,-)$], $F_{mn} = \partial_m A_n - \partial_n A_m$ is the stress tensor of the gauge field, and the parameter l has the dimensionality of a length and characterizes the magnitude of the anomalous magnetic moment. The quantity A_m has the dimensionality l^{-1}, while the gauge coupling constant e (which is "concealed" in A_m) has the dimensionality $l^{-1/2}$. In $D=2+1$, the γ^m satisfy $\gamma^m\gamma^n = g^{mn} - i\epsilon^{mnl}\gamma l$, so Eq. (1) can be rewritten as

$$(\partial_m + iA_m + i\frac{l}{2}\epsilon_{mnl}F^{nl})\gamma^m\Psi = im\Psi(x). \tag{2}$$

In order to deal with the interaction which stems from the anomalous magnetic moment of the particle in $D=2+1$, we must therefore add to A_m the dual vector $F_m=\epsilon_{mnl}F^{nl}$. This procedure can also be carried out for scalar fields $\Phi(x)$, for which the equation of motion becomes

$$(\partial_m+iA_m+i\frac{l}{2}F_m)^2\Phi(x)=-m^2\Phi(x). \tag{3}$$

An equation of this type arises in, for example, the generalized dynamics of relativistic particles which is found by the twistor approach.[1]

It is interesting to compare the energy spectrum of spinor and scalar particles with an anomalous magnetic moment in an external electromagnetic field. For example, the energy spectrum of a particle with an anomalous magnetic moment in a uniform external magnetic field H is

$$E=[m^2-(2n+1-2s)H]^{1/2}+lH, \qquad n=0,1,2,...,$$

where s is the helicity of the particle, which appears in the definition of the normal magnetic moment. The latter is zero for a scalar particle.

The action describing the interaction of field $\Phi(x)$ with the Chern–Simons–Maxwell Abelian field is

$$S=\int d^3x[|(\partial_m+iA_m+i\frac{l}{2}F_m)\Phi|^2-V(|\Phi|^2)+\theta\epsilon^{mnl}A_m\partial_nA_l-\frac{1}{4e^2}(F_{mn})^2], \tag{4}$$

where we have used a Higgs potential $V(|\Phi|^2)$. The field Φ has a dimensionality $l^{-1/2}$; the dimensionless parameter θ is proportional to the ratio of the topological mass and the coupling constant of the gauge field.[2]

The equations of motion for the Chern–Simons–Maxwell field which follow from (4) are symmetric with respect to the dual gauge vector F^m and with respect to the Noether matter current J^m:

$$\theta F^m+(-1/(2e^2))\epsilon^{mnl}\partial_nF_l=J^m+l\epsilon^{mnl}\partial_nJ_l. \tag{5}$$

When the vacuum expectation value of the field Φ is nonzero ($\langle\Phi\rangle=\mu$), and the gauge symmetry is spontaneously broken (this case actually corresponds to a minimum of the Hamiltonian of the theory in the long-wave approximation, to which we are restricting the present paper), the effective action for a vector gauge field becomes

$$S=\int d^3x\left[-\left(\frac{1}{4}-\frac{l^2}{2}\mu^2e^2\right)F_{mn}^2+e^2(\theta+l\mu^2)\epsilon^{mnl}A_m\partial_nA_l+(e\mu)^2A_mA^m\right], \tag{6}$$

where we have redefined the vector field: $A_m\to A_m/e$. Analysis of action (6) yields the following conclusions.

The contribution to the Maxwell term leads to a "renormalization" of the coupling constant.

If there is no Chern–Simons term in the original action ($\theta = 0$), such a term arises as a result of the spontaneous symmetry breaking. (The generation of a Chern–Simons term by radiative corrections in a theory with a spontaneously broken parity was studied in Ref. 3.)

There are two critical points for μ^2, the parameter of the spontaneous symmetry breaking; at these points, the dynamics of the vector field changes substantially. One of these points corresponds to $\mu^2 = 1/(2l^2e^2)$. At it, the Maxwell kinetic term disappears from action (6), and a theory analyzed previously[4] arises. At $\mu^2 > 1/(2l^2e^2)$, the Maxwell term acquires the wrong sign, so the theory loses its unitarity in this sector.

At the other critical point ($\mu^2 = -\theta/l$), the Chern–Simons term disappears, and we are left with the ordinary action for a massive vector field. At this point, there is a "spin-flip": Since there is no correlation between the signs of θ and l, the vector field will have helicities of different signs, depending on whether the conditions $\mu^2 > -\theta/l$ or the condition $\mu^2 < -\theta/l$ holds.

Away from the critical points, action (6) describes states of a vector field which have opposite helicities and different masses.[5]

We must stress that a model based on action (4) should be regraded as an effective theory, valid in the long-wavelength approximation and at small values of l.

It would be interesting to see to what extent this model or some modification of it (e.g., one incorporating terms with higher powers of l) could serve as the foundation of a systematic quantum theory.

It would also be interesting to study the spin-statistics properties of the field $\Phi(x)$ in connection with the possible appearance of anyons, to search for vortex solutions, and to search for possible applications in solid state physics.

We wish to thank D. V. Volkov, A. A. Zheltukhin, I. V. Kriva, and Yu. P. Stepanovskiĭ for useful discussions.

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Translated by D. Parsons