Stochastic resonance in an all-optical passive bistable system

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The first reported observation of a stochastic resonance in an all-optical passive bistable system is described. The appearance of a stochastic resonance in the case of a weak signal and a noise is explained in the theory of a linear response.

The concept of a stochastic resonance was introduced by Benzi *et al.*[1-3] to explain the periodic occurrence of glacial stages on the earth. The term has subsequently been extended[4] to label the increase and subsequent decay of the signal-to-noise ratio R at the output of a system as the noise intensity rises. The ratio R is understood as the ratio of the spectral fluctuation intensity $Q(\omega)$ at the frequency of the external field, Ω, to the corresponding spectral intensity in the absence of a field, $Q^{(0)}(\Omega)$. Although this phenomenon is of a general nature and is independent of the specific physical nature of the bistable system, it has so far been studied primarily by numerical and analog modeling.[5-7] The only "real" physical system in which a stochastic resonance has been observed to date is a ring dye laser with competing modes.[4]

In this letter we are reporting the first observation of a stochastic resonance in an all-optical passive bistable system. Specifically, we studied a three-mirror system of coupled resonators.[8,9] The first resonator is formed by a membrane consisting of a thin film (≈ 1 μm thick) of a semiconducting GaSe single crystal, separated by a metal diaphragm ≈ 500 μm in diameter from a plane dielectric mirror. The air-filled gap between the mirror and the membrane is ≈ 10 μm wide and forms a second resonator. The incident beam from an argon laser, of intensity I and wavelength $\lambda = 514.5$ nm, propagates along the normal to the mirror. The intensity I is modulated in time by an electrooptic shutter, to which a regular signal and also a noise with a frequency $\leqslant 100$ kHz are applied. The optical bistability (Fig. 1a) arises because of the thermoelastic sag of the membrane caused by the laser beam. A distinctive feature of this system is that it has lumped parameters: The film sags as a whole, and transverse effects have no influence on the dynamic behavior of the film. A multistability and self-oscillations have been demonstrated previously in this system.[8,9] Fluctuation effects have also been studied.[10] In particular, when the intensity I of the incident light is modulated by a Gaussian noise of intensity D,

$$I = \bar{I} + \delta I(t) + A\cos(\Omega t), \qquad < \delta I(t) >= 0,$$

$$< \delta I(t)\delta I(t') >= \frac{D}{\tau_c}\xi\left[\frac{|t-t'|}{\tau_c}\right], \qquad \int_0^\infty \xi(x)dx = 1, \tag{1}$$

the spectral fluctuation intensity in the transmitted light, $I_T(t)$, i.e.,

$$Q(\omega) = \lim_{t_o\to\infty}\frac{1}{4\pi t_0}\left|\int_{-t_0}^{t_0} dt \exp(i\omega t) I_T(t)\right|^2, \tag{2}$$

contains a Lorentzian peak at a zero frequency:

$$Q_0(\omega) = \frac{w_1 w_2}{\pi}(I_{T1} - I_{T2})^2(W_{12} + W_{21})/[\omega^2 + (W_{12} + W_{21})^2]. \tag{3}$$

Here W_{ij} are the probabilities for $i \to j$ transitions between the stable states which coexist at the given $\bar{I}$; $w_{1,2}$ are the populations of the states ($w_1/w_2 = W_{21}/W_{12}$); and $I_{T1,2}$ are the values of I_T in the stable states.

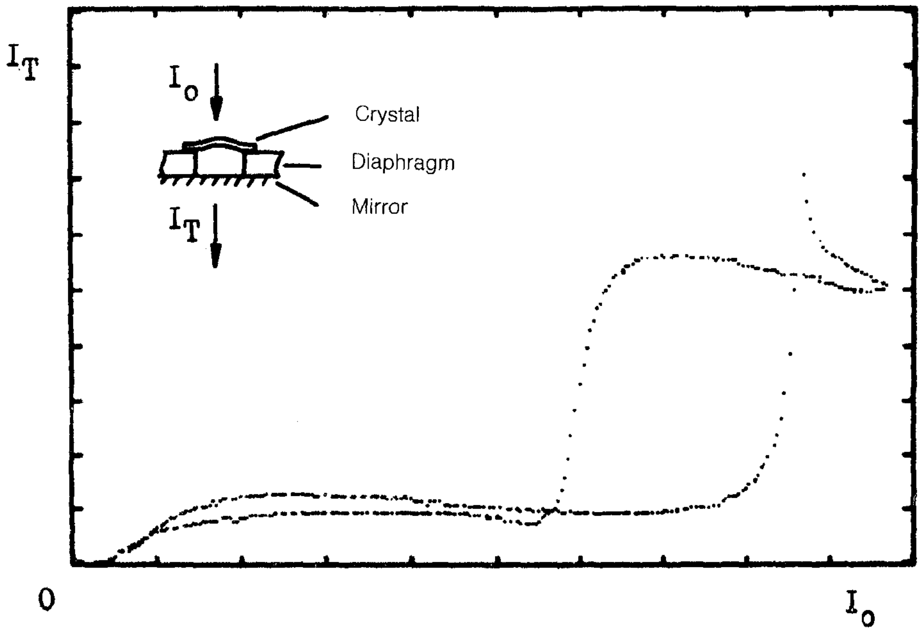

FIG. 1. Hysteresis loop in the light transmitted by the three-mirror system (the element is shown schematically in the inset).

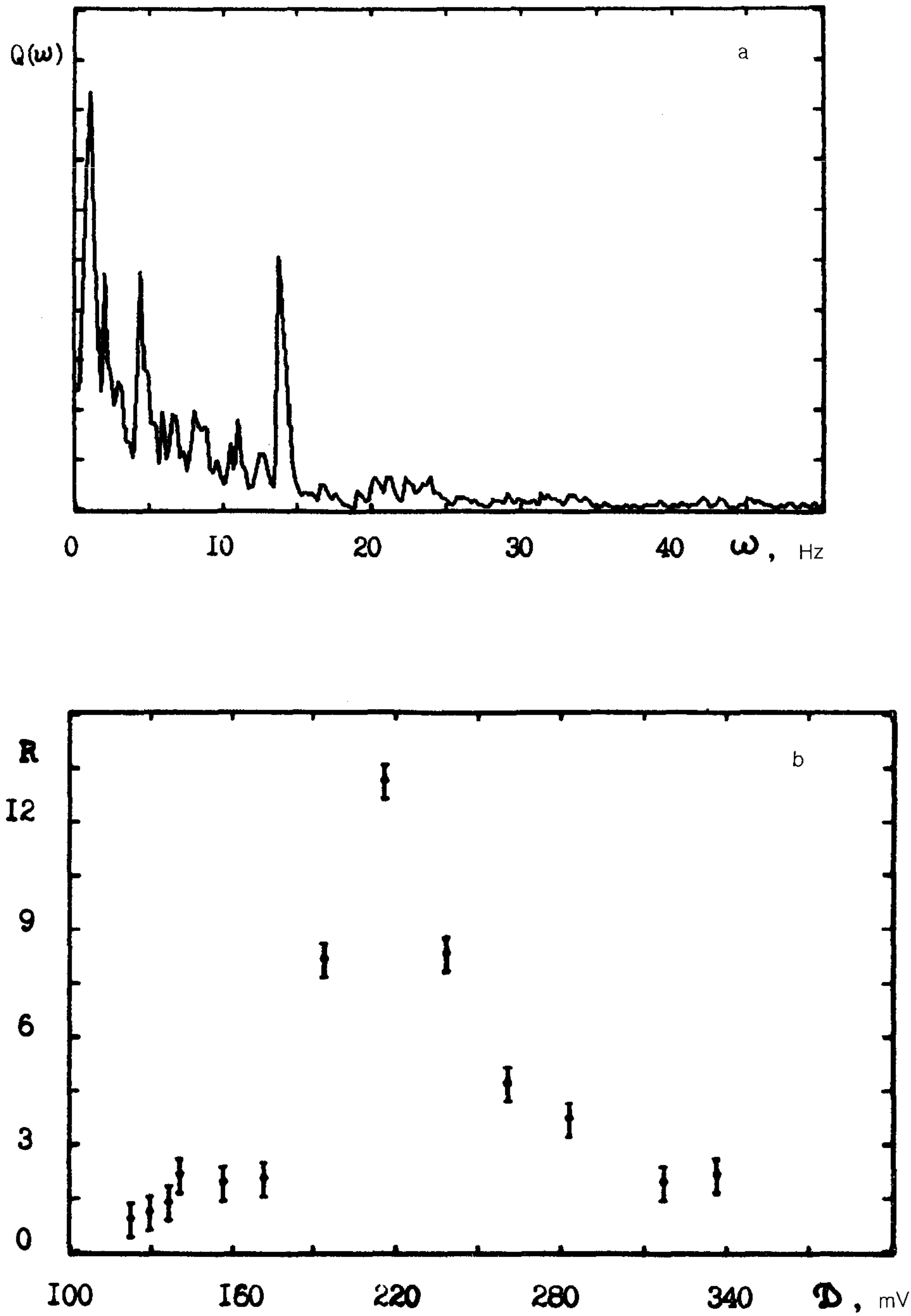

FIG. 2. a—Spectral fluctuation intensity in the light leaving the system for a signal-to-noise ratio of 7; b—stochastic resonance.

If the noisy modulation is accompanied by a periodic modulation of the incident light, i.e, if a term $A\cos(\Omega t)$ is added to I [see (1)], a δ-function peak appears in the spectral fluctuation intensity at the frequency $\pm\Omega$. The intensity of this peak can be calculated by using the Debye approximation to describe the relaxation of the phase shift of the light, φ, in the air-filled gap of the resonator. This phase shift is determined by the thermoelastic sag of the membrane:

$$\dot{\varphi}+\frac{1}{\tau}\Delta\varphi=IM(\varphi),\quad \varphi=\varphi_0+\Delta\varphi,\quad I_T=IN(\varphi), \tag{4}$$

where the 2π-periodic functions $M(\varphi)$ and $N(\varphi)$ are, in this normalization of I, the absorption and transmission coefficients of the system. Assuming that the noise $\delta I(t)$ is a broad-band (white) noise, $\tau_c \ll \tau$, we can show without difficulty that under the condition $\Omega\tau \ll 1$ the intensities of the δ-function peaks in the spectral fluctuation intensity at the frequencies $\pm\Omega$ (the "signal") are equal to $(1/4)A^2|K|^2$ in the weak-modulation limit, where

$$K=\sum_{n=1,2}{}' w_n\frac{dI_{Tn}}{dI}+\frac{w_1w_2}{D}(I_{T1}-I_{T2})\frac{W_{12}+W_{21}}{W_{12}+W_{21}+i\Omega}\int_{\varphi_1}^{\varphi_2}d\varphi M^{-1}(\varphi). \tag{5}$$

In general, the spectral fluctuation intensity at low frequencies is a superposition of a δ-function peak at the frequency Ω, the zero-frequency peak described by (3), and a broad, smoothly varying pedestal (at $\omega\tau \ll 1$), which is proportional to D and which is small if D is small. Since the probabilities for the fluctuational transitions, $W_{ij} \propto \exp(-R_i/D)$, increase with the noise intensity in accordance with an activation law, the signal-to-noise ratio also increases sharply with D if the heights of the "potential barriers" satisfy $R_{1,2} \gg D$, as is clear from (3)–(5). The subsequent decay of the stochastic resonance at large D is determined by a growth of the pedestal (not by a growth of the zero-frequency peak) and by an abrupt slowing of the growth of W_{ij} at $R_i \gg D$.

This particular effect has been observed experimentally (Fig. 2). A sinusoidal signal at a frequency of 14 Hz was applied to an electrooptic modulator along with a noisy signal. The intensity of the transmitted signal was detected by a digital oscilloscope on line with a microcomputer. The signal-to-noise ratio in the spectral fluctuation density increased sharply beginning at a certain value of the noise amplitude D and then fell off with a further increase in D (Fig. 2).

In summary, a stochastic resonance has been demonstrated for the first time in an all-optical passive bistable system, in a case of a weak sinusoidal signal and a multiplicative noise. The effect has been explained on the basis of the theory of a linear response.

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