Time-varying diffraction effects in the propagation of an electromagnetic pulse in vacuum

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(Submitted 23 January 1991)
Pis'ma Zh. Eksp. Teor. Fiz. **53**, No. 4, 188–191 (25 February 1991)

A new optical effect is predicted: As an electromagnetic field pulse with a steep leading edge propagates in vacuum, the transverse distribution and strength of the field at the leading edge remain constant. This effect is linked with the existence of a time-dependent "vacuum dispersion." An analogy is drawn between this effect and Sommerfeld and Brillouin's problem of the propagation of a signal in a dispersive medium and also with the coherent propagation of a small-area pulse (a 0π pulse) in a gas of resonantly absorbing two-level particles.

The influence of diffraction effects on the propagation of beams of quasimonochromatic waves in the case in which the spatial distribution of the field can be described in terms of slowly varying amplitudes is presently under study. In this approximation the diffraction is interpreted as a diffusion of the complex amplitude of the beam in the transverse direction.[1] The approximation of slowly varying amplitudes, however, cannot be used to describe the diffraction propagation of an electromagnetic field pulse whose length is only a few wavelengths (as few as one). We show below that the temporal structure of the field pulse becomes a governing factor in the evolution of the spatial distribution of this pulse: It becomes necessary to speak in terms of the appearance in the diffraction of dispersive effects. Such effects did not arise in an analysis of the diffraction of quasimonochromatic waves by virtue of the very formulation of the problem.

The effects involved here can be summarized as follows. In a wave with a nonzero angular spectrum, photons whose wave vector makes a large angle with the wave direction (and which are thus comparatively influential in causing diffraction of the wave) have a relatively small velocity projection onto this direction and are delayed with respect to the leading edge of the wave by a relatively large amount. The spatial distribution of the beam at a given instant is thus determined by the dynamic rebunching of the photons at all preceding times. As a result of this "vacuum dispersion," the problem of the diffraction of the pulse becomes somewhat analogous to Sommerfeld and Brillouin's problem of the propagation of a signal with a sharp leading edge in a medium with a time-dependent dispersion.[2]

1. An electromagnetic field pulse is propagating along the z axis in vacuum. The angular spectrum of the pulse can be characterized by the parameter $\mu = L_{\parallel}/L_{\perp} \ll 1$, where $L_{\parallel}$ and $L_{\perp}$ are length scales of the variation of the pulse field in the longitudinal and transverse directions, respectively. The wave equation for the field of the pulse,

$$\frac{\partial^2 \mathcal{E}}{\partial z^2} + \Delta_{\perp} \mathcal{E} - \frac{1}{c^2}\frac{\partial^2 \mathcal{E}}{\partial t^2} = 0, \qquad (1)$$

where $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$, can be simplified since μ is small. The space-time variations of the field $\mathscr{E}(\vec{r},t)$ stem from a rapid transport of the pulse along the characteristic $\tau = t - z/c$ and a slow variation of its shape along the longitudinal and transverse coordinates, so a solution of Eq. (1) can be sought in the form $\mathscr{E}(\vec{r},t) = \mathscr{E}(z,\vec{r}_\perp,\tau)$. Taking into account the slow variation of the pulse shape along the z direction, we find from (1) the equation

$$\frac{\partial^2 \mathcal{E}}{\partial z \partial \tau} - \frac{1}{2} c \Delta_\perp \mathcal{E} = 0. \tag{2}$$

for a field which is a quasimonochromatic wave we find from (2) a parabolic diffraction equation for the complex amplitude of this wave. Integrating (2) over τ, we find

$$\frac{\partial \mathcal{E}}{\partial z} - \frac{1}{2} c \Delta_\perp \int_{-\infty}^{\tau} \mathcal{E}(\tau') d\tau' = 0. \tag{3}$$

According to (3), in terms of the real field $\mathscr{E}$ the diffractive variation in the shape of the pulse is determined by the transverse field distribution not only at the given instant but also at all preceding times. One might say that a transverse variation of the field gives rise to an inertial "response of vacuum."

For the pulse energy $W = c/4\pi \int\int dx dy \int_{-\infty}^{\infty} d\tau \mathscr{E}^2$ we find from (3)

$$\frac{dW}{dz} = -\frac{1}{2} c \int\int dx dy \left\{ \left[\frac{\partial}{\partial x} \Psi(\vec{r}, \infty) \right]^2 + \left[\frac{\partial}{\partial y} \Psi(\vec{r}, \infty) \right]^2 \right\}, \tag{4}$$

where $\Psi(\vec{r},r = \int_{-\infty}^{r} \mathscr{E}(z,\vec{r}_\perp,\tau^2) d\tau'$.

Energy is conserved because the positive expression in the integrand vanishes: The functions $c(\partial/\partial x)\Psi(\vec{r},\tau)$ are the and $c(\partial/\partial y)\Psi(\vec{r},\tau)$ longitudinal components of the electric and magnetic fields. They vanish as the pulse passes $(\tau \to \infty)$.

It thus follows that $\Psi(\vec{r},\infty)$ does not depend on the transverse coordinates. It also follows that since we have $\mathscr{E} \to 0$ in the limit $|\vec{r}_\perp| \to \infty$, we have $\Psi(\vec{r},\infty) = 0$. The area under the electric field of the pulse is thus zero at each spatial point.

(2) Let us examine the analogy between diffractive projection and the propagation of a pulse through a medium of two-level particles under conditions corresponding to a coherent interaction.[3,4] Expanding $\mathscr{E}$ in the transverse waves $\sim \exp\{i\vec{\kappa}\vec{r}_\perp\}$, we find the following equation from (3) for the component $\mathscr{E}_{\vec{\kappa}}(z,\tau)$:

$$\frac{\partial \mathcal{E}_{\vec{\kappa}}}{\partial z} = -\frac{1}{2} c \kappa^2 \int_{-\infty}^{\tau} \mathcal{E}_{\vec{\kappa}}(\tau') d\tau'. \tag{5}$$

The solution of (5) is a 0π pulse, i.e., a variable-sign oscillating function which covers an area $\int_{-\infty}^{\infty} \mathscr{E}_{\vec{\kappa}}(z,\tau) d\tau = 0$ at each spatial point. This result means that the condition $\Psi(\vec{r},\infty) = 0$ found above for the field of the diffracting pulse can be interpreted as the formation of a 0π pulse in a coherently absorbing medium.

3. As an example we consider the diffractive propagation of a pulse with an

abrupt field increase at the plane leading edge:

$$\mathcal{E}(0,\vec{r}_\perp,\tau)=\begin{cases}\mathcal{E}(\vec{r}_\perp), & \tau\geq 0\\ 0, & \tau<0,\end{cases} \tag{6}$$

where $\mathscr{E}(\vec{r}_\perp)$ is the transverse field distribution. The change in the shape of the pulse as a result of diffraction is described by

$$\mathcal{E}(z,\vec{r}_\perp,\tau)=\int\int \mathcal{E}(\vec{\kappa})e^{i\vec{\kappa}\vec{r}_\perp}J_0(\sqrt{2\kappa^2 cz\tau})d\kappa_x d\kappa_y, \tag{7}$$

where $\mathscr{E}(\vec{\kappa})$ is the Fourier transform of the initial distribution $\mathscr{E}(\vec{r}_\perp)$, and $J_0(\xi)$ is the Bessel function of index 0. It follows immediately from (7) that the leading edge is transported directly without distortion: $\mathscr{E}(z,\vec{r}_\perp,0)=\mathscr{E}(0,\vec{r}_\perp,0)$. The situation is completely analogous to the coherent propagation of a pulse with a steep leading edge through a medium of two-level particles.[3]

Another aspect of solution (7) is that over propagation distances $z>L_\perp^2/L_\parallel$, where $L_\parallel$ is a longitudinal length scale of the field variation behind the pulse front, the leading part of the beam transforms into a subpulse. The length of the pulse decreases in proportion to the distance traversed (z) because of a diffractive erosion of the field at the trailing edge of the pulse. The leading edge of the subpulse retains the initial transverse structure precisely: The field strength at $\tau=0$ does not vary with the propagation distance z (Fig. 1). This subpulse, propagating at the velocity of light, might be called a "diffractive precursor," since all known diffraction effects (the conversion of a plane wave into a divergent wave, the corresponding decay of the field with the propagation distance, etc.) develop behind it.

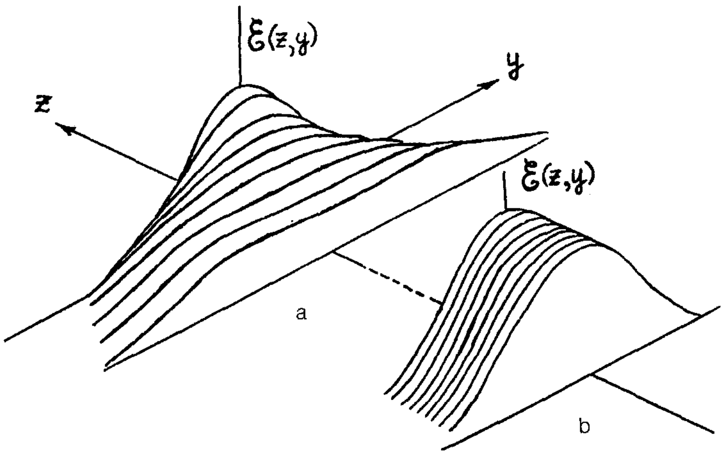

FIG. 1. Evolution of an electromagnetic wave with a steep leading edge as it propagates along the z axis. a—Initial field distribution near the leading edge; b—formation of a diffractive precursor as a result of the wave propagation.

It follows from this discussion that the diffractive precursor is somewhat analogous to the Sommerfeld precursor which arises at a steep leading edge of a pulse in a medium is associated with the finite time which it takes the medium to respond to an external field, in our case the role of the inertial response of the vacuum is played by the spreading of the field in the transverse direction. The inertia of this process (with respect to the leading edge of the pulse) is essentially a consequence of the finite velocity of light in vacuum: The field at the plane leading edge cannot be diffracted, since the leading edge itself is moving at the velocity of light. A transverse spreading of the field would thus be possible only behind the leading edge.

[1]L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, Pergamon, Oxford, 1984.
[2]A. Sommerfeld, *Optics* [Russian translation] IL, Moscow, 1953; L. Brillouin and M. Parodi, *Wave Propagation in Periodic Structure* [in French], 1955.
[3]M. D. Crisp. Phys. Rev. **A1**, 1604 (1970).
[4]E. M. Belenov and A. V. Nazarkin, Pis'ma Zh. Eksp. Teor. Fiz. **51**, 252 (1990) [JETP Lett. **51**, 288 (1990)].

Translated by D. Parsons