A new class of normal Fermi liquids

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The topological structure of the Green's function of the Fermi liquid is analyzed. This structure does not change upon transition to a marginal Fermi liquid or a Luttinger Fermi liquid. It changes, however, upon transition to a state such as the fermion condensate, in which the Fermi surface spreads into a Fermi band.

The discussion of the origin of high-T_c superconductivity has recently revived interest in the uncommon states of the Fermi liquid.[1] In addition to the standard Fermi liquid with a clearly defined pole, marginal Fermi liquids,[2] in which the residue of Z vanishes, have been analyzed as a logarithmic function of the frequency $[Z \sim 1/\ln(\omega_c/\omega)]$ in the Green's function near the Fermi surface

$$G(\omega,\vec{p}) \approx \frac{Z}{\omega - v_F(p-p_F) + i\gamma(p)\mathrm{sign}(p-p_F)} .$$

This situation occurs when the interaction of electrons with transverse photons is taken into account.[3] There is also the Luttinger Fermi liquid in which Z decays in a power-law fashion.[4] The latter is found in one-dimensional Fermi systems, but it apparently can also be found in systems of higher dimensionality.[5] Another type of Fermi liquid, called a fermion condensate, has been suggested in Ref. 6 for a strong interaction of fermions. In the random-phase approximation the energy $\varepsilon(\vec{p})$ of the Fermi quasiparticles, which is reckoned from the chemical potential, vanishes not at the surface $p = p_F$, as in the ordinary Fermi system, but over the entire momentum band $p_1 < p < p_2$. In other words, the Fermi surface spreads out over the entire "Fermi band" (the fermion condensate).

We will discuss the general topological structure of the Green's function for these systems and we will show that the Luttinger Fermi liquid and the marginal Fermi liquid belong to the same topological class as the ordinary Fermi liquid, i.e., their Green's function has a vortex singularity in the momentum space and the locus of points where this characteristic is situated forms the Fermi surface. In the fermion condensate state the vortex singularity diffuses into a band of finite width (a vortex sheet), which, by analogy with the vortices in the superfluid, corresponds to the splitting of the vortex with one circulation quantum into two half vortices which are linked by a vortex sheet.

We will consider a single-particle Green's function $G(\Omega,\vec{p})$ on an imaginary semiaxis of frequencies $\omega = i\Omega$. According to the general analytical properties, $G(\Omega,\vec{p})$ can have singularities only at $\Omega = 0$ (Ref. 7). In the normal Fermi liquid these singularities are situated on the Fermi surface and are characterized by the invariant

$$N = \mathrm{tr} \oint_C \frac{dl}{2\pi i} G(\Omega,\vec{p}) \partial_l G^{-1}(\Omega,\vec{p}) \ , \tag{1}$$

where *tr* is the trace of the spin or band indices of the Green's function, and the integral is taken over an arbitrary contour C in the space $(\Omega,\vec{p})$ which encloses the singularity. In Fig. 1, we show for simplicity a 2D Fermi liquid, where the contour C encloses the linear singularity — the Fermi line. The singularity is analytical in nature, whereas N agrees with the multiplicity of the pole in the singularity. In the ordinary Fermi liquid, for example, the Green's function has a first-order pole in the singularity, $G(\Omega,\vec{p}) \sim Z/(z - v_F p_F)$, where $z = v_F p - i\Omega$. Consequently, $N = 1$ for each of the two spin projections, so the total charge $N = 2$. It is important that the invariant N remains an integer even when the singularity is not of a polar nature. This index describes a shift in the phase Φ of the Green's function as the contour is traversed. Since this phase shift amounts to $2\pi N$, it cannot change continuously and so it is conserved in the case of small changes in the Green's function. In this respect, it is analogous to the topologically stable singularities of the superfluid condensate phase. It can be said that the Fermi line in Fig. 1 in this analogy corresponds for one spin component to the quantized vortex line with a circulation quantum $N = 1$. [In the case of a 3D Fermi liquid the quantum vortex in a four-dimensional space $(\vec{p},\Omega)$ forms a 2D surface — a Fermi surface; for a 1D Fermi liquid the vortex is a point in a 2D (p,Ω) space].

The invariant N remains constant when an ordinary Fermi liquid is replaced by nonpolar (marginal or Luttinger) Fermi liquids. The Green's function for a 1D Luttinger spinless Fermi liquid has the following form[5] near each of the two Fermi points $\pm p_F$: $G(\omega,k = p \pm p_F) \sim (v^2k^2 - \omega^2)^g/(\omega \pm vk)$. Extending G analytically to the imaginary semiaxis, where $G(\Omega,p) \sim (z - v_F p_F)^{g-1}(z^* - v_F p_F)^g$, and calculating the integral (1), we find, as in the ordinary case, the same vortex singularity for $z = v_F p_F$ with $N = 1$, regardless of the value of g. This also holds for the marginal Fermi liquid[2,3] with $G \sim [i\Omega \ln(i\Omega) - v_F(p - p_F)]^{-1}$. Switching to marginal and Luttinger

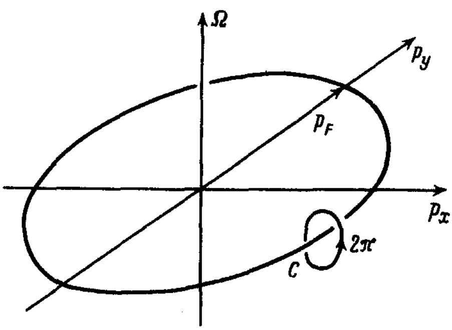

FIG. 1. Fermi surface in a 2D (3D) Fermi liquid is a topological stable special vortex line (a surface) in a three-dimensional (four-dimensional) space Ω, p_x, p_y (Ω, p_x, p_y, p_z). Upon traversal of this line the phase of the Green's function changes by 2π. Such a structure remains the same in marginal and Luttinger Fermi liquids.

states thus does not change the topological structure of the Green's function of the Fermi liquid, preserving the concept of the Fermi surface as the surface of the vortex singularities of the Green's function. The situation does not change when the spin structure is taken into account. The integral (1) for the ordinary Fermi liquid in this case gives the total charge $N=2$ for the two spin components, and the most that can happen upon switching to nonpolar Fermi liquids is the splitting of the spin-degenerate Fermi surface with $N=2$ into two well-defined Fermi surfaces, each with $N=1$, which correspond to holons and spinons.[1] From the topological point of view, this is equivalent to the splitting of the Fermi surface by the Pauli magnetic field which separates the fermion energies with different spin projections, which also gives rise to nondegenerate Fermi surfaces with $N=1$.

The situation is entirely different in the fermion condensate system analyzed in Ref. 6. The vortex line with $N=1$, which cannot disappear because of the topological stability, expands to a vortex sheet (Fermi band; see Fig. 2 for the dimensionality $D=2$) on which the phase Φ of the Green's function changes abruptly. In the random-phase model,[6] in which the energy $\varepsilon(\vec{p})=0$, and hence $G=1/i\Omega$ in the Fermi band $p_1<p<p_2$, the phase Φ changes by a constant, equal to π, upon passage through $\Omega=0$. This means that the boundaries of the vortex sheet are vortices with a half-integer with an integer circulation quantum $N=1/2$ since the phase shift around them amounts to π. This is a crude feature, i.e., it remains even in an exact solution of equations for the Green's function, which generally does not agree with the result obtained from the model. the exact solution has not yet been found, although it can be assumed that in the case of a sufficiently small splitting p_2-p_1, when the Green's function behaves identically near each half-vortex, it is a simple power function

$$G(\vec{p};\Omega)=\frac{Z}{(z-v_F p_1)^{1/2}(z-v_F p_2)^{1/2}}, \tag{2}$$

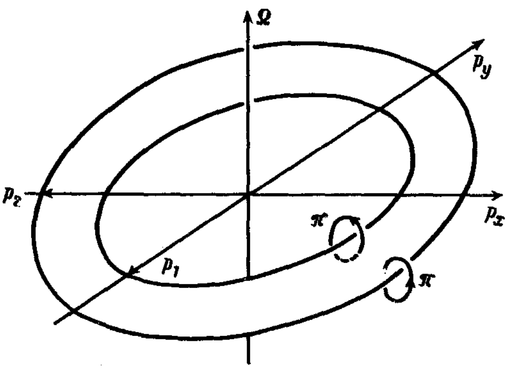

FIG. 2. In a fermion condensate the Fermi liquid spreads into a Fermi band, whose boundaries $p=p_1$ and $p=p_2$ are half-quantum vortices. Upon traversal of such a half quantum, the phase of the Green's function changes by an amount π.

which corresponds to the square-root cut in the interval $p_1 < p < p_2$: the phase Φ of the Green's function differs by an amount π on each side of the cut.

In contrast with the marginal and Luttinger Fermi liquids, the system with a fermion condensate is, in terms of its topological structure, an essentially new class of Fermi liquids. The change from the Fermi surface in the ordinary Fermi liquid to a Fermi band in a fermion condensate is linked with a change in the topological characteristic of the Green's function and is a modification of the Lifshitz topological phase transitions which occur at zero temperature. This phenomenon can occur not only in a normal Fermi liquid but also in superconductors, where a sufficiently strong pairing interaction could lead to a detection of the Fermi surface even in the superconducting state.[8] The Fermi surface would be a surface of singularities in the expanded Green's function of a superconductor (the Gor'kov functions).

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