The geometrical phase and resonance conversion of the neutrino spin

A. Yu. Smirnov
Institute of Nuclear Research, USSR Academy of Sciences, 117312 Moscow

(Submitted 20 February 1991)
Pis'ma Zh. Eksp. Teor. Fiz. **53**, No. 6, 280–282 (25 March 1991)

It is shown that the geometrical phase arising in neutrino propagation in a magnetic field can induce resonance spin conversion: $\nu_L \to \nu_R$. The application of this effect to solar neutrinos is studied.

The appearance of a geometrical (topological) phase[1] in the precession of a neutrino spin in a magnetic field is related to the rotation of the field $\vec{B}$ along the path of the neutrino in the transverse plane.[2-4] The geometrical phase can significantly affect the solar neutrino flux.[2-4] Here we discuss a new effect—resonance conversion of the neutrino spin induced by the geometrical phase.

Let us consider a system of right- and left-handed neutrinos, $\vec{\nu}_S = (\nu_R, \nu_L)$ with magnetic moment μ evolving in matter and a transverse magnetic field. Let the field rotate along the neutrino path in the transverse plane: $\vec{B} = Be^{i\Phi}$, where $\Phi(t)$ is the rotation angle. The evolution equation for $\vec{\nu}_S$ has the form

$$i\frac{d\vec{\nu}_S}{dt} = \hat{H}\vec{\nu}_S, \quad \hat{H} = \begin{bmatrix} V/2 & \mu Be^{-i\Phi} \\ \mu Be^{i\Phi} & -V/2 \end{bmatrix}, \tag{1}$$

where V is the splitting of the ν_L and ν_R energy levels due to the difference between how these components interact with matter and, in general, due to their mass difference:

$$V = \sqrt{2}G_F n^{eff} - \frac{\Delta m^2}{2E}. \tag{2}$$

Here G_F is the Fermi constant, $\Delta m^2 = m^2(\nu_L) - m^2(\nu_R)$, E is the neutrino energy, n^{eff} is the effective concentration of particles with which the neutrino interacts, $n^{eff} = (n_e - n_n)$ for the system $(\nu_{eL}, \bar{\nu}_{\mu R})$ and $n^{eff} = (n_e - n_n/2)$ for (ν_{eL}, ν_{eR}), and n_e and n_n are the electron and neutron concentrations, respectively. The imaginary part of $\hat{H}$ is eliminated by the transformation $\vec{\nu}_S = \hat{U}\vec{\nu}'_S$, where $\hat{U} = \mathrm{diag}\{e^{-i\Phi/2}, e^{i\Phi/2}\}$, and the equation of motion for $\vec{\nu}'_S$ has the form

$$i\frac{d\vec{\nu}'_S}{dt} = \begin{bmatrix} (V-\dot{\Phi})/2 & \mu B \\ \mu B & -(V-\dot{\Phi})/2 \end{bmatrix}\vec{\nu}'_S \tag{3}$$

$\dot{\Phi} = d\Phi/dt$ is the rate of change of the geometrical phase. Since $\hat{U}$ is diagonal, the transition probabilities for $\vec{\nu}_S$ and $\vec{\nu}'_S$ are identical.

For constant parameters V (i.e., n^{eff}), B, and $\dot{\Phi}$ the system (3) can be solved analytically. The survival probability (the transition $\nu_L \to \nu_L$) is equal to $P = \overline{P} + (A_P/2)\cos(2\pi t/l_P)$, where A_P is the precession depth

$$A_P = \sin^2 2\theta_m = \frac{(2\mu B)^2}{(V - \dot{\Phi})^2 + (2\mu B)^2}, \tag{4}$$

θ_m is the ν'_R and ν'_L mixing angle, l_P is the precession length

$$2\pi l_P^{-1} = [(V - \dot{\Phi})^2 + (2\mu B)^2]^{1/2}, \tag{5}$$

and $\overline{P} = 1 - A_P/2$ is the averaged probability. Equations (4) and (5) are the generalization of the results of Ref. 3 to the case of precession in matter.

Let us consider the dependence of the precession depth on $\dot{\Phi}$. For $\dot{\Phi} \gg V$ it can be written as $A_P = \dot{\Phi}^2_{\rm dyn}/(\dot{\Phi}^2 + \dot{\Phi}^2_{\rm dyn})$, where $\dot{\Phi}_{\rm dyn} = 2\mu B$ is the rate of change of the ordinary dynamical phase. If $\dot{\Phi} \gg \dot{\Phi}_{\rm dyn}$ (the case considered in Ref. 4), the depth is strongly suppressed: $A_P \simeq (\dot{\Phi}_{\rm dyn}/\dot{\Phi})^2 \ll 1$. Because of this circumstance, the $\nu_L \to \nu_R$ transition probability is small, although the total precession phase is much larger than the dynamical phase and possibly reaches π: $\Phi_{\rm pr} = 2\pi t/l_P \simeq \dot{\Phi} t \simeq \pi$. In Ref. 4 the dependence of A_P on $\dot{\Phi}$ was omitted and the statement that the geometrical phase makes it possible to solve the solar neutrino problem for small fields and small μ is incorrect.

For $V = \dot{\Phi}$ or explicitly

$$\sqrt{2} G_F n^{eff} - \frac{\Delta m^2}{2E} = \dot{\Phi} \tag{6}$$

the mixing (4) becomes maximal, and therefore (6) is the resonance condition (level crossing).[5] A qualitatively new case is realized for $\Delta m^2 = 0$, i.e., for Dirac or Zel'dovich–Konopinskiĭ–Makhmud neutrinos (the latter are particularly interesting in connection with gauge theories giving large μ). In this case for $\dot{\Phi} = 0$ there is no level crossing in the Sun: $n^{eff} > 0$, the matter suppresses precession.[6] However, a change of the geometrical phase, $\dot{\Phi} \neq 0$, can cancel the effect of the matter and the resonance condition is satisfied. Therefore, for monotonically varying density of the medium (and $\dot{\Phi}$ which does not change too rapidly) resonance spin conversion $\nu_L \to \nu_R$ will occur. The resonance condition (6) for $\Delta m^2 = 0$ is independent of the energy and is realized only for a definite direction of rotation of the field: $\dot{\Phi} > 0$. This asymmetry is related to the difference between the ν_L and ν_R interactions. The conversion efficiency depends on the degree of adiabaticity. The adiabaticity condition,[5] which ensures the most complete transition, has the following form at the resonance ($V = \dot{\Phi}$):

$$\frac{2(2\mu B)^2}{\sqrt{2} G_F \dot{n}^{eff} - \ddot{\Phi}} \gg 1. \tag{7}$$

For $\dot{n}^{eff} < 0$ negative $\ddot{\Phi} < 0$ improves the adiabaticity; for $\dot{\Phi} = 0$ the dependence of the condition (7) on the phase vanishes.

Let us make some estimates of the Sun. The rotation of $\vec{B}$ along the neutrino path can be related to the fact that in the convective zone the magnetic field lines form spirals wrapped around a torus.[3] If the total rotation angle for a neutrino intersecting the torus is $\Delta\Phi \simeq \pi$ (which is realized for a dense spiral) along the path $\Delta R = 0.2R_\odot$ ($R_\odot$ is the radius of the Sun), then $\dot{\Phi} \simeq \Delta\Phi/\Delta R = 2.2\times 10^{-10}$ cm^{-1} and the resonance density, according to (6), is equal to 0.06 g/cm^3. Similarly, for $\Delta\Phi = \pi/4$ we obtain 0.015 g/cm^3. Layers with such densities are located at depths of $0.2R_\odot$ and $0.1R_\odot$, where the field can be fairly strong (6). For constant Φ the conversion effect due to the geometrical phase coincides with that due to spin-flavor conversion[7] for $\Delta m^2/2E = \dot{\Phi}$. The corresponding probabilities are $P_{\rm geom}(\Phi) = P_{\rm s-f}(E/\Delta m^2 = 1/2\dot{\Phi})$, where $P_{\rm geom}$ is independent of E.

For $\Delta m^2 \neq 0$ the geometrical phase shifts, according to (6), the resonance of the spin-flavor precession[7] in energy, either by $\Delta m^2/E$ or the density.

Another application of this effect is to neutrinos from gravitational collapse.

The author is grateful to Ya. A. Smorodinskiĭ, who in 1986 first pointed out the possibility of topological phase effects in a system of mixed neutrinos.

[1] M. Berry, Proc. Roy. Soc. London Ser. A **392**, 45 (1984).
[2] J. Wudka and J. Vidal, Preprint UCD-88-40, University of California, Davis (1988).
[3] C. Aneziris and J. Schechter, Preprint SU-4228-449, Syracuse University, Syracuse (1990).
[4] J. Vidal and J. Wudka, Phys. Lett. **259B**, 473 (1990).
[5] S. P. Mikheev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; A. Yu. Smirnov, in *Proc. of the Conf. "Neutrino-88,"* edited by T. Kafka *et al.*, Boston, 1988, p. 123.
[6] M. B. Voloshin and M. I. Vysotskiĭ, Yad. Fiz. **44**, 845 (1986) [Sov. J. Nucl. Phys. **44**, 544 (1986)]; M. B. Voloshin, M. I. Vysotskiĭ, and L. B. Okun', Zh. Eksp. Teor. Fiz. **91**, 754 (1986) [Sov. Phys. JETP **64**, 446 (1986)].
[7] C.-S. Lim and W. J. Marciano, Phys. Rev. D **37**, 1368 (1988); E. Kh. Akhmedov, Phys. Lett. **213B**, 64 (1988).

Translated by Patricia Millard