Fractional fermions and a phase transition in Fermi gas

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It is demonstrated that a phase transition caused by the creation of fractional fermions occurs in a dense Fermi-gas at low temperatures. Corrections associated with the interaction between fractional fermions are analyzed.

In certain field theory models, solitons acquire a fractional fermion charge[1,2] due to interaction with fermions. Fractional fermions exist due to the zeroth bound level in the fermion spectrum. The fermion number of a soliton with an occupied zeroth level is 1/2 and the fermion number of a soliton with a free zeroth level is $-1/2$. A phase transition associated with the restoration of spontaneously broken symmetry occurs in a Fermi-gas at high densities.[3,4] The present paper demonstrates that an additional phase transition associated with the creation of fractional fermions exists before the symmetry restoration point.

We consider a $(1+1)$-dimensional model with a Lagrangian[1]

$$L = \frac{1}{2}(\partial\varphi)^2 - \frac{\lambda}{4}(\varphi^2 - c^2)^2 + \bar{\psi} i\hat{\partial}\psi - g_F\bar{\psi}\varphi\psi. \quad (1)$$

This model has a soliton solution

$$\varphi_s = \pm c\,\mathrm{th}(m_H x/2), \quad m_H = \sqrt{2\lambda}c, \quad M = \frac{2}{3}m_H c^2. \quad (2)$$

In a dense Fermi gas ($\mu \gg m_F$) at low temperatures ($T \ll m_F, m_H$) the fermion number of the soliton is determined by the contribution of the zeroth bound level:

$$n_F^s = \frac{1}{2} + \frac{2}{\pi}\frac{m_F^2}{m_H \mu} \simeq \frac{1}{2}, \qquad m_F = g_F c, \quad (3)$$

where μ is the chemical potential of the fermions.

It follows from Eq. (3) that it would be energy efficient for a fermion of energy $\epsilon = \mu_{cr} = 2M$ to produce a pair of solitons with $n_F^s = 1/2$ and to drop to the zeroth level. The density at which the fermions reach such energies is $n_{cr} = \mu_{cr}/\pi$. Any further compression of the Fermi gas will create and expand the number of soliton pairs. The Fermi levels in the continuum are no longer filled beginning at $n = n_{cr}$, and the chemical potential ceases to grow with density: $\mu = \mu_{cr} =$ const. This behavior suggests that a phase transition is present and a new Fermi-gas phase has appeared. In this case, the gas density remains constant: the gas no longer "flexes."

An increase in the soliton density (n_s), however, causes the N_s-fold degenerate zeroth level to split due to the interaction between the solitons. A thin energy band arises in place of the level with $\epsilon = 0$. This band has a width

$$\Delta \simeq m_F \exp\{-m_F/n_s\}. \tag{4}$$

Equations (2) and (3) are valid for $m_F \ll m_H c_1$, $c \gg 1$, and Eq. (4) is valid for $m_F \ll m_H$ and is slightly modified for $m_H \ll m_F \ll m_H c$ (details will be published at a later date). With $m_F \gtrsim m_H c$, the perturbation theory in g_F is not valid, and Eqs. (2)–(4) become meaningless.

The free energy density of the system at zero temperature is

$$\frac{1}{L} F(T=0) \simeq \left(M_s + \frac{\Delta}{2}\right) n_s + \frac{1}{2}\pi (n_{cr} + \Delta n)^2, \tag{5}$$

$$n_s = 2[n - (n_{cr} + \Delta n)], \qquad \Delta n \simeq \frac{\Delta}{\pi}.$$

From this expression we can easily obtain expressions for the chemical potential and gas pressure, which take into account the interaction between solitons:

$$\mu = \frac{1}{L}\frac{\partial F}{\partial n} = \mu_{cr} + \Delta, \tag{6}$$

$$p = -\frac{\partial F(N)}{\partial L} \simeq p_{cr} + 2M\frac{\Delta}{\pi}, \qquad p_{cr} \simeq \frac{1}{2}\pi (n_{cr})^2. \tag{7}$$

At soliton densities $n_s \sim m_F$, where $\Delta \lesssim m_F$, the zeroth level band begins to overlap the continuum, so that fermion creation is no longer energy efficient. Hence the chemical increases gradually, beginning at $\mu \simeq \mu_{cr} + m_F$ up to the symmetry recovery point $\mu_{cr}^{(0)}$. The number of solitons that are produced therefore is $n_s \sim m_F$.

Fractional fermions also exist in $(3+1)$-dimensional theory with a scalar field triplet φ^a and a fermion doublet ψ from the $SU(2)$ group[1]

$$L = -\frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2}(D_\mu \varphi^a)^2 - \frac{\lambda}{4}(\varphi^2 - c^2)^2 + i\bar{\psi}_n(\hat{D}\psi)_n - g_F \bar{\psi}_n T^a_{nm} \varphi^a \psi_m. \tag{8}$$

The soliton of this model is the 'T Hooft–Polyakov monopole[6,7] of mass

$$M \simeq \frac{4\pi m_v}{g_v^2}, \qquad m_v = g_v c.$$

We assume that $m_F \ll m_H \ll m_v$. The fermion number of the monopole in a dense Fermi gas ($\mu \gg m_F$) at low temperatures ($T \ll m_F$) then turns out to be

$$n_F^M \simeq \frac{1}{2} + \frac{1}{6\pi}\frac{m_v}{\mu} + \frac{8}{\pi}\frac{g_F^2}{\lambda}\frac{\mu}{m_v}. \tag{9}$$

It is thus clear that at rather high densities ($\mu \gg m_v$) the fermion number of the monopole is determined, as in the $(1+1)$-dimensional case, by the zeroth bound level: $n_F^M = 1/2$.

A phase transition occurs as a result of fermion "fracturing" when $g_F \ll g_0\sqrt{\lambda}$, $\mu_{cr} = 2M \ll \mu_{cr}^{(0)}$, and just before the dissolution of the monopoles in the medium.

With $\mu = \mu_{cr} = 2M$ a cold Fermi gas thus undergoes a phase transition, where the derivatives of the chemical potential and gas change abruptly. The resulting phase is an intermediate phase between the phases $\langle \varphi \rangle = c$, $m_F = g_F c$ and $\langle \varphi \rangle = 0$, $m_F = 0$.

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