Effect of impurity spin dynamics on weak localization

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The relaxation of paramagnetic impurity spins under conditions of weak localization is shown to give rise to a term linear in temperature $[\sigma(T)-\sigma(o)]=(T/T_s)(e^2/4\pi h)(D\tau_s)^{(2-d)/2}$ at temperatures in the range $T<T_s=\epsilon_F n_s/n_e$.

1. Weakly localized corrections often play a dominant role in the temperature and magnetic field dependences of the conductivity of dirty metals (with $\epsilon_F\tau/h>1$) at low temperatures.[1] Since they have a quantum nature and since they result from the interference of electron waves propagating in different directions of the same diffusion trajectory, they are extremely sensitive to any invariance violations, $t\Rightarrow -t$, in the system. The presence in the conductor of paramagnetic impurities (of concentration n_s) that experience an exchange interaction with the conduction electrons, $V_{int}=(\mathbf{Ss})\gamma\delta(\mathbf{x}-\mathbf{x}_0)$, will cause a spin relaxation of such electrons and will contribute to the cutoff of the trajectory lengths that produce the interference pattern.[2] In addition, spin-flip scattering of intensity $\tau_s^{-1}\sim(S+1)S\gamma^2 n_s/(\lambda_F^d\epsilon_F\hbar)$ at the lowest temperatures and in zero magnetic field is the primary factor that determines the coherence length. (*H*ere we assume $\tau_s\gg\tau$).

In this paper we consider the possible effect of paramagnetic impurity spin dynamics on weakly localized corrections to the conductivity. The dynamics are understood to represent the Korringa relaxation[3] of localized spins. This relaxation results from the same spin–spin interaction with thermalized free electrons and occurs in a time $\tau_K^{-1}\sim T\gamma^2/(\lambda_F^{2d}\epsilon_F^2\hbar)$.

2. Let us first determine the case in which such dynamics is important. We recall that electron diffusion trajectory pairs whose transit time is less than the spin relaxation time τ_s, are the only pairs that contribute to interference effects. The spin dynamics of the scatterers will therefore manifest itself only when the Korringa time τ_K is shorter than the one-electron time τ_s:

$$T\gamma^2/(\lambda_F^{2d}\epsilon_F^2\hbar)\sim\tau_K^{-1}>\tau_s^{-1}\sim\gamma^2 S(S+1)n_S/(\lambda_F^d\epsilon_F\hbar). \quad (1)$$

This condition determines the temperature $T_s\propto\epsilon_F n_s/n_c$ at which the static limit (at $T\ll T_s$) of the coherence breakdown due to scattering by the paramagnetic impurities is replaced by the dynamic limit (at $T\gg T_s$).

The difference between these two limits is determined by the circumstance whether or not the electron, which transverses a closed circuit in opposite directions, encounters a magnetic impurity in the same spin state ($\tau_K>\tau_s$) or in completely uncorrelated states ($\tau_K < \tau_s$). In the first case, the correlation between the pairs of electrons that traverse the same loop in opposite directions decays at different rates in the singlet channel ($\tau_{s,0}^{-1} = 2\tau_s^{-1}$) and the triplet channel[2] ($\tau_{s,1}^{-1} = 2/3\tau_s^{-1}$). In the second case the rate of coherence breakdown is degenerate with respect to the total spin of the pair and is exactly equal to the inverse relaxation time of a single electron, $\tau_{s,0}^{-1} = \tau_{s,1}^{-1} = \tau_s^{-1}$. This difference produces a quantitative difference in the weakly localized corrections to the conductivity under these two conditions.

3. We employ an expression of the quantum corrections to the conductivity in terms of a two-particle Green's function in the particle–particle (cooperon) channel for an analytical description of this effect. In the temporal representation we have

$$\sigma_q = (e^2/2\pi h)\int_\tau^\infty dt[C_0(t,-t;\vec{x},\vec{x}) - 3C_1(t,-t;\vec{x},\vec{x})]. \tag{2}$$

The equation for the cooperon is derived by summing ladder diagrams[1,2] and in our specific case is different from the familiar expression in that it accounts for Korringa relaxation in the correlation of the impurity spin states $S_\alpha(0)S_\beta(t) > = \delta_{\alpha\beta}S(S+1)\exp\{-|t|/\tau_K\}$. As a result, the equations for the single component $C_0(t,t';x,x')$ and the triplet component $C_1(t,t';x,x')$ of the cooperon can be written as follows:

$$\{\partial_t - D\nabla^2 + \tau_s^{-1}[1 + c_J\exp(-|t|/\tau_K)]\}C_J(t,t';x,x') = \delta(t-t')\delta(x-x'), \tag{3}$$

where $c_0 = 1$, $c_1 = -1/3$, and the substitution of the solutions of this equation into Eqs. (2) in the conductors of different dimensions gives

$$\sigma_{KB} = (e^2/2\pi h)(D\tau_s)^{(2-d)/2}\int_{\tau/\tau_s}^\infty \frac{d\theta}{\theta^{d/2}}(\exp\{-2\theta - 2\frac{\tau_K}{\tau_s}[1-e^{-\theta\tau_s/\tau_K}]\}$$

$$-3\exp\{-2\theta + \frac{2\tau_K}{3\tau_s}[1-e^{-\theta\tau_s/\tau_K}]\}). \tag{4}$$

It follows from this expression that the quantum corrections in the conductivity in the static limit ($\tau_K^{-1} = 0$) and the dynamic limit ($\tau_K^{-1} \gg \tau_s^{-1}$) for the paramagnetic impurity spins differ by

$$\Delta\sigma = \sigma(\tau_K^{-1} \Rightarrow \infty) - \sigma(\tau_K^{-1} = 0)$$

$$= \frac{e^2}{2\pi h}(D\tau_s)^{(2-d)/2}\begin{cases} 0,5(3^{3/2} - 1 - 2^{3/2})\pi^{1/2} = 1,2, & d=1 \\ \ln(27/4) = 1,9, & d=2 \\ 4,65, & d-3 \end{cases} \tag{5}$$

in samples of different dimensions. Specifically, in thin films ($d = 2$) this difference is a universal quantity, $\Delta\sigma = (e^2/2\pi h)\ln(27/4)$.

4. The transition of Eq. (1) from the static limit to the dynamic limit in spin impurity behavior can be observed from the temperature dependences of the conductivity at temperatures $T < 4T_s$. As can be seen from Eq. (4), after the substitution $\tau_s/\tau_K = T/T_s$, impurity spin relaxation in this temperature range leads to a term linear in T:

$$\sigma(T) - \sigma(0) = \frac{e^4}{2\pi h}(D\tau_s)^{(2-d)/2}\frac{T}{T_s}[2^{d-6} + (3/4)^{(d-6)/2}]\Gamma(3 - d/2)$$

in the conductivity, and at $T \gg T_s$ it becomes saturated [Eq. (5)].

An important condition for the manifestation of this temperature dependence of the conductivity in the limit $T \Rightarrow 0$ is that the Korringa relaxation must be faster than the inelastic phase relaxation of the conduction electrons [otherwise, condition (1) cannot hold in the paramagnetic limit $\tau_\varphi > \tau_s$]. Often the primary mechanism is a phase shift of the electrons due to the soft e-e collisions,[2] while the temperature dependence of the conductivity described above in d-dimensional electron systems can be expected in samples with $(\gamma n_e/\epsilon_F)^2(n_e L_s^d)^{(d-2)/2}(l/\lambda_F) \gg 1$, i.e., in thin films or three-dimensional conductors.

5. In conclusion we note that an analogous anomaly can also occur in the conductivity of systems with strong spin-orbit scattering, $\tau_{sO} < \tau_s$, in which the only contribution to the quantum correction to the conductivity in Eq. (2) comes from the singlet component of the cooperon, while $\Delta\sigma = e^2/(2\pi h)\ln 2$ is of the same order of magnitude and has the same sign as in the preceding case.

It should be pointed out that at the lowest temperatures, $T \ll T_s$, the spin dynamics may produce a certain nonohmic behavior.

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