$SL(2,Z)$ invariance in a closed supermembrane

S. D. Odintsov
Tomsk State Pedagogical Institute

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It is suggested that $SL(2,Z)$ is an invariance group of the amplitudes of a closed supermembrane against a plane background. The vacuum energy of a closed supermembrane compactified on a torus can indeed be represented in a modulus-invariant form.

One of the major problems of string theory today is that of seeking the correct invariance group (hidden symmetry).[1] The role played by modular invariance [$SL(2,Z)$ invariance] in the theory of closed strings is well known. The representation of the string amplitudes in a modulus-invariant form makes it possible to avoid summations over an infinite number of equivalent integration regions; a single fundamental region is sufficient. The requirement of modular invariance of the partition function off the mass shell leads to equations for the background fields which are equivalent to the equations which arise from the condition for the vanishing of the β functions.

Similar questions should evidently arise in membrane theory (see, for example, Refs. 2 and 3 for reviews), which has received far less study. Since a string may be thought of as a form of a reduced membrane, we think it would be natural to suggest that the amplitudes of a closed membrane on a plane background might be represented in a modulus-invariant form.

Let us consider a $D=11$ closed supermembrane on a plane background s_1 (temperature)$\times R_{11-d}\times T_d$. Bytsenko and Ktitorov[5] have calculated the vacuum energy in this theory through a semiclassical quantization with $d=0$; Bytsenko and Odintsov[6] have done the same for $d\neq 0$. (On a plane background R_{11}, the vacuum energy vanishes by virtue of supersymmetry.) We write the vacuum energy V as the triple integral $\int_{-(1/2)}^{1/2} ds \int_{-(1/2)}^{1/2} d\tau_1 \int_0^\infty d\tau_2$ of some expression. We introduce the new variable $\tau=\tau_1+i\tau_2$. It is not difficult to verify that the integrand in the vacuum energy from Refs. 5 and 6 is invariant under $\tau\to\tau+1$ [the discrete translation group U, consisting of matrices of the type $U\equiv\left(\begin{smallmatrix}1 & b\\ 0 & 1\end{smallmatrix}\right), b\in Z$).] The group U is a Borel subgroup of the $SL(2,Z)$ group. We can then use the theorem of Ref. 4, which states that if some expression is invariant under certain subgroups of $SL(2,Z)$ (in particular, U), then it can be represented in a modulus-invariant form. The recipe for actually finding this representation is extremely simple.[4]

As an example we consider the vacuum energy in a $D=11$ supermembrane against an S_1 (temperature)$\times R_8\times T_2$ background.[6] A modulus-invariant expression for this energy can be written in the form[1)]

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$$V = -\int_F \frac{d^2\tau}{\pi^2 R_1 R_2} \sum_{(c,d)=1} (2\mathrm{Im}\gamma_{cd}\tau)^{-11/2} [\Theta_3(0| \frac{i\beta^2}{2\mathrm{Im}\gamma_{cd}\tau})$$

$$-\Theta_4(0| \frac{i\beta^2}{2\mathrm{Im}\gamma_{cd}\tau} \int_{-1/2}^{1/2} dy \sum_{\substack{n_1, n_2, \\ k_1, k_2 = -\infty}}^{\infty} \exp\{-\frac{(n_1 R_1 n_2 R_2)^2}{2\pi} \mathrm{Im}\,\gamma_{cd}\tau$$

$$-(\frac{n_1^2}{R_1^2} + \frac{n_2^2}{R_2^2}) \frac{\mathrm{Im}\,\gamma_{cd}\tau}{2\pi} + 2\pi i(n_1 k_1 \mathrm{Re}\gamma_{cd}\tau + n_2 k_2 y))\}$$

$$\times [\prod_{m_1, m_2 = -\infty}^{\infty} \frac{1-\exp(-\frac{\mathrm{Im}\gamma_{cd}\tau}{\pi}\omega_{m_1,m_2} + 2\pi i m_1 \mathrm{Re}\gamma_{cd}\tau + 2\pi i y m_2)}{1+\exp(-\frac{\mathrm{Im}\gamma_{cd}\tau}{\pi}\omega_{m_1 m_2} + 2\pi i m_1 \mathrm{Re}\gamma_{cd}\tau + 2\pi i y m_2)}]^{-8},$$

where $F = \{-\frac{1}{2} \leqslant \tau_1 \leqslant \frac{1}{2}, \tau_2 > \sqrt{1-\tau_1^2}\}$ is the fundamental region, R_1 and R_2 are the radii of the torus, $\Theta_3(a,b) - \Theta_3$ is the Jacobi function, $\omega_{m_1 m_2} = [(m_1 n_1 R_1)^2 + (m_2 n_2 R_2)^2]^{1/2}$, $\Sigma_{(c,d)=1}$ is the sum over all mutually simple c and d, $\gamma_{cd} = (\begin{smallmatrix} * & * \\ c & d \end{smallmatrix})$, and the asterisk $(*)$ means that any transformation in $SL(2,Z)$ with given (c,d) can be utilized as a representative of a coset expansion.[4] Other amplitudes can be written in a modulus-invariant form in the same way as above. Interestingly, the amplitudes of supersymmetric closed p-branes against such a background can be represented in a form which is invariant under $G = [SL(2,Z)]^{n+1}$ for $p = 2n+1$ or $G = [SL(2,Z)]^{n+1} \otimes U$ for $p = 2n+2$.

A question remains open here: Is there a wider invariance group of closed supermembrane which contains $SL(2,Z)$ as a subgroup?

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[1)] The details of these calculations will be reported in an expanded version of this paper.

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