Instability, oscillations, and chaos of polarization state of self-intersecting light beam

O. P. Zaskal'ko, A. G. Kamushkin, and V. V. Korobkin
Institute of General Physics, Academy of Sciences of the USSR

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During the self-intersection of a light beam accompanied by a rotation of the polarization plane, possibly to a state orthogonal to the initial state, in a medium with a cubic, scalar, reactive, nonlinear susceptibility, the polarization goes from a steady state into states of periodic and random oscillations.

Polarization multistability and chaos are the subjects of active research because of the possible development of optical logic devices which could operate without an energy loss. Many optical systems which operate on the basis of optical nonlinearities of various symmetries have been studied (see the review by Zheludev[1]). Among the systems which exhibit a polarization chaos, the one which we have proposed appears to be the simplest to set up and the simplest in terms of the nonlinearity involved. It turns out that in a medium with an instantaneous scalar nonlinear response, a copropagating self-intersection of a light beam accompanied by a rotation of polarization plane in a feedback loop causes the polarization state to become unstable and to undergo a transition to chaos.

Let us consider a slab of an optically isotropic medium whose nonlinear polarizability can be written in the form[2] $\vec{P}^{(3)} = A(\vec{E}^*\cdot\vec{E})\vec{E}$, where A is real. We assume that a monochromatic plane wave $\vec{E}_0\exp(i\vec{k}\vec{r} - i\omega t)$ is incident on the slab in the $z=0$ cross section and propagates through the medium at an angle θ from the z axis. After passing through the medium, the wave is returned by an external optical line, in which the polarization plane of the wave is rotated. The wave then propagates through the medium again, from the same side as before, but this time at an angle $-\theta$ from the z axis.

The change in polarization state is conveniently analyzed in a form which is invariant under the choice of a Cartesian coordinate system, through the use of Stokes parameters:[3]

$$\xi_1 = (E_xE_y^* + E_x^*E_y)/I, \quad \xi_2 = i(E_x^*E_y - E_xE_y^*)/I, \quad \xi_3 = (|E_x|^2 - |E_y|^2)/I,$$

where $I = |E_x|^2 + |E_y|^2$. The set of three numbers (ξ_1, ξ_2, ξ_3) determines a unit vector: the Stokes vector $\vec{\xi}$. Figure 1 shows the relationship between the polarization state and the corresponding orientation of the Stokes vector. We denote by $\vec{\xi}\,'$ and $\vec{\xi}\,''$, respectively, the Stokes vector of the light wave in its first and second passages through the medium. Using $\vec{P}^{(3)} = A(\vec{E}^*\cdot\vec{E})\vec{E}$, we then find the following expression from Maxwell's equations:

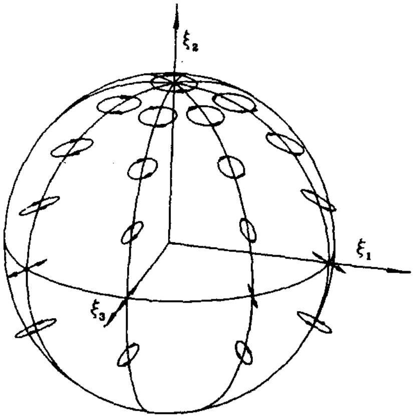

FIG. 1. Poincaré sphere showing the correspondence of the direction of the Stokes vector and the polarization state of the light wave.

$$\left(\frac{\partial}{\partial z}+\frac{n}{c}\frac{\partial}{\partial t}\right)\xi' = -\left(\frac{\partial}{\partial z}+\frac{n}{c}\frac{\partial}{\partial t}\right)\xi'' = \left(\frac{2\pi A I \omega}{cn\cos\vartheta}\right)\cdot \xi' \times \xi''. \tag{1}$$

The boundary conditions are written in the form $\xi'(0,t) = \xi^0, \xi''(0,t) = \hat{\rho}\xi'(l,t-T)$, where $\hat{\rho}$ is the matrix of the polarization change in the space of Stokes vectors, and T is the time delay of the light wave in the feedback loop. For the Stokes vector of the light wave after the first passage through the medium, we find

$$\vec{\xi}'(l,t) = \vec{\xi}^0 + \vec{m}(t)\times[\vec{\xi}^0\times\vec{m}(t)](\cos(|\vec{M}(t)|\eta)-1) + [\vec{\xi}^0\times\vec{m}(t)]\sin(|\vec{M}(t)|\eta), \tag{2}$$

where $\vec{M}(t) = \vec{\xi}^0 + \hat{\rho}\vec{\xi}'(l,t-T)$, $\vec{m}(t) = \vec{M}(t)/|\vec{M}(t)|$, and $\eta = (2\pi A\omega/cn)Il$.

Let us examine in more detail the case in which the initial wave is linearly polarized, and the polarization plane is rotated through 90° in the feedback loop. For this case we have

$$\hat{\rho} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

In this formulation of the problem, waves with orthogonal polarizations obviously do not interact with each other; i.e., system (1) has a unique steady-state solution, which corresponds to an unperturbed polarization state $\vec{\xi} = \vec{\xi}^0 = (0,0,1)$, $\vec{\xi}''(0,0,-1)$. We restrict the discussion to a discrete mapping of the Stokes vectors with a step T. A numerical analysis shows that a steady state is stable only if $\eta < 1$. At $\eta = 1$, a "hard" spontaneous violation of the polarization symmetry occurs: The unperturbed state becomes unstable and decays into a cycle with a period of $2T$. The direction along which the system leaves the unperturbed state depends on small initial fluctuations of the polarization. As the intensity is raised further, there is a cascade of period doublings (Fig. 2b), with an index which tends toward the Feigenbaum constant $\delta \approx 4.6$, according to a determination from the first even bifurcations. The time evolution of the polarization then becomes random.

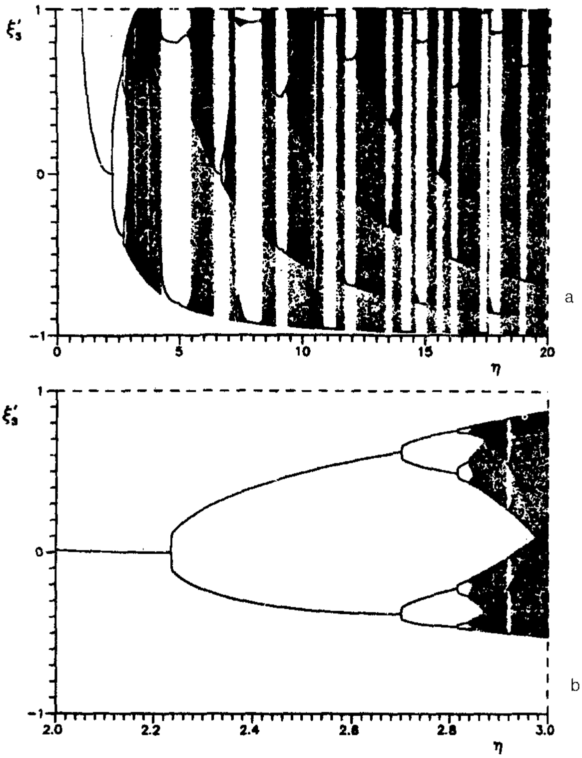

FIG. 2. The projection ξ_3' of the Stokes vector of the light after the first passage through the slab of medium onto the direction of the Stokes vector at the entrance to the medium in a simulation of an adiabatic increase in the intensity.

At higher values of η, "windows" corresponding to cyclic states reappear in the chaos. These windows in turn undergo a period-doubling transition to chaos (Fig. 2a). The polarization fluctuations in the chaos may be confined or may propagate over essentially the entire region of permissible values. "Crises in the chaos" (sharp changes in the dimensions of a random attractor) are observed. The region of values of the Stokes vector (the attractor) on the Poincaré sphere is, in the case of a cyclic change in polarization state, a family of circles whose centers lie on the ξ_3 axis (Fig. 3a). These circles are formed by states with a fixed ξ_3 projection which are precessing at various rates. The Stokes vector jumps among these states in a cyclic manner. In the case of a random behavior, the attractor consists of bounded regions (Fig. 3b).

A similar behavior has been observed for other types of polarization transformations in the feedback loop. This behavior is found in a long list of real physical devices which transform the polarization of optical radiation without an intensity loss (a Fresnel rhombus, rotating quartz, and a λ/N phase plate), for which the transformation matrix $\hat{p}$ corresponds to rotations around various axes in Stokes-vector space. A totally different situation arises if a polarizer is placed in the feedback loop. In this case, (first) the region of regular behavior becomes significantly larger, since the polarizations of both beams are fixed at the entrance to the nonlinear medium, and (second) the light intensity at the exit is not preserved.

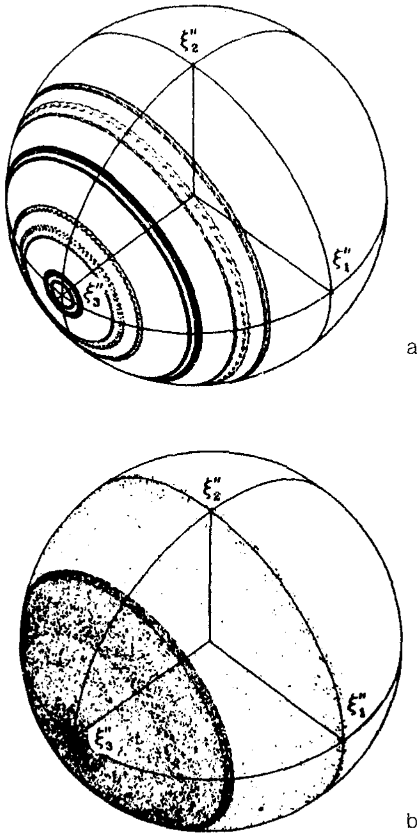

FIG. 3. Examples of (a) a cyclic attractor at $\eta = 2.84$ and (b) a random attractor at $\eta = 3.0$ for the Stokes vector of the light leaving the medium.

This formulation of the problem is also of interest in connection with problems of phase conjugation in arrangements with a self-intersection of the pump beam.[4] The self-effect of the pump beam is a parasitic process in such cases and can lead to a distortion and an instability of the wavefront. In media with a scalar response, a necessary condition for the absence of such a self-effect is that the polarizations of the wave during the first and second passages be orthogonal. It follows from the present study, however, that a simple rotation of the polarization plane in a feedback loop is not sufficient here: Even at comparatively low intensities, corresponding to a nonlinear phase shift on the order of unity, an absolute instability of the polarization state of the self-intersecting light beam occurs.

[1]N. I. Zheludev, Usp. Fiz. Nauk **157**, 683 (1989) [Sov. Phys. Usp. **32**, 357 (1989)].
[2]I. R. Shen, *Principles of Nonlinear Optics*, Nauka, Moscow, 1989.
[3]L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, Nauka, Moscow, 1979 (an earlier edition was translated into English by Pergamon, New York, 1976).
[4]I. M. Bel'dyugin *et al.*, Kvant. Elektron. (Moscow) **12**, 2394 (1985) [Sov. J. Quantum Electron. **15**, 1583 (1985)].

Translated by D. Parsons