Temperature-gradient instability caused in plasma by conducting end surfaces

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In a plasma with open field lines and an electron-temperature gradient in the direction transverse with respect to the magnetic field, a rapid instability and an elevated transverse transport of plasma can occur if the plasma is bounded along the field lines by highly conducting material surfaces. This result may be of interest for open systems and also for such entities as the earth's magnetosphere (in which case the ionosphere would play the role of the underlying surface) and the solar atmosphere (in which case the photosphere would play this role).

An instability stems from a potential difference between the plasma and conducting end surfaces of the device which are equipotential surfaces. Numerous experiments in open systems have solidly established that a potential difference on the order of a few times T_e/e (T_e is the electron temperature, and e the electron charge) is set up between the end surfaces and the center of the plasma. The instability is thus particularly sensitive to specifically a gradient of the electron temperature. Insulating ends (this limiting case is achieved in the laboratory by using multisection ends) are no longer equipotentials, and the instability disappears. This result, along with the results of a recently published paper,[1] shows that the widespread opinion that conducting ends tend to oppose the instability of a plasmas on open field lines is by no means always correct. An earlier discussion of certain aspects of how a potential difference between the plasma and the ends affects plasma stability can be found in Refs. 2 and 3, among other places.

A curvature of magnetic field lines is not a necessary condition for the instability. To analyze the instability, we thus examine the plane model shown in Fig. 1. To get a better picture of the instability, we consider it in its "pure" form, ignoring effects which stem from the gradient of the ion temperature and density. The only parameter which varies along the magnetic field is the electron temperature T_e.

We assume that the planes which bound the plasma ($z = \pm L/2$ in Fig. 1) reflect most of the incident ions, so the ion absorption coefficient ϵ is much less than unity. For an open system, the approximation $\epsilon \ll 1$ is fairly good in simulating the presence of end systems which suppress the loss of plasma (e.g., strong mirrors in an open gasdynamic system).[4] The results found below remain qualitatively valid at $\epsilon \sim 1$. The condition $\epsilon \ll 1$ guarantees that in the unperturbed state the plasma properties are uniform along the field lines within terms $\sim \epsilon$.

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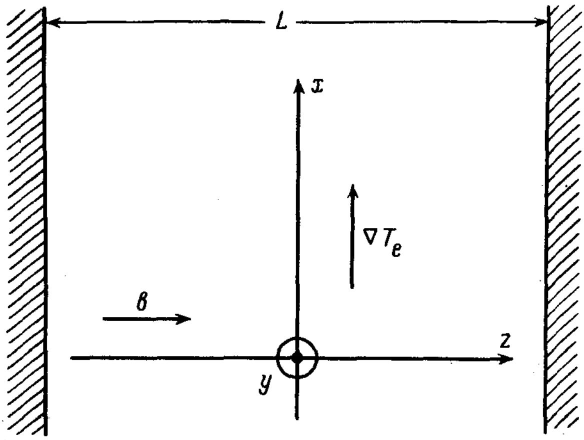

FIG. 1. Geometry of the problem. The magnetic field is uniform and is directed along the z axis. The hatched regions are conducting walls which bound the plasma in the direction along the magnetic field.

With regard to the complex instability frequency ω, we assume that $|\omega|$ satisfies the condition $v_{Ti} \ll |\omega| L \ll v_{Te}$, where $v_{Te,i} = (2T_{e,i}/m_{e,i})^{1/2}$ (m_e and m_i are the masses of the electron and the ion). The condition $|\omega| L \ll v_{Te}$ allows us to assume that the electrons have a Boltzmann distribution along the field lines.

The motion in the plasma volume is described by the equations of ideal MHD. We consider perturbations $f(z)\exp(-i\omega t + ik_x x + ik_y y)$, which are localized along x and y, with $k_y \gg k_x \gg a^{-1}$, [$a \equiv (d \ln T_e/dx)^{-1}$ is the length scale of the gradient in the electron temperature], and which vary smoothly along z. Since the instability can be very rapid, with frequencies exceeding v_A/L, where $v_A = B/\sqrt{4\pi n m_i}$ is the Alfvén velocity, we allow for perturbations of the magnetic field [although the plasma pressure is assumed to be weak: $\beta \equiv 8\pi n(T_e + T_i)/B^2 \ll 1$]. These perturbations would be associated with a possible variation of the displacement of the force tube, ξ_x, along z.

From the MHD equations for a perturbation $\xi_x(z)$, which is even with respect to the $z=0$ plane (for brevity, we ignore odd perturbations), we easily find

$$\xi_x(z) = \xi^* \frac{\cos(\Omega z/v_A)}{\cos(\Omega L/2v_A)}, \tag{1}$$

where $\xi^* = \xi_x(z = L/2)$, $\Omega = \omega - k_y v_E$, and v_E is the unperturbed electric drift velocity. We can express the perturbation of the current to the end, $\delta j_\parallel$; the potential perturbation at the boundary of the Debye sheath (on the plasma side), $\delta\varphi$; and the perturbation of the electron temperature near the wall, δT_e in terms of ξ^*:

$$\delta j_\parallel = i\xi^* \frac{k_y c \Omega B}{4\pi v_A} \operatorname{tg} \frac{\Omega L}{v_A}, \quad \delta\varphi = \xi^* \frac{\Omega B}{k_y c}, \quad \delta T_e = -\xi^* \frac{dT_e}{dx}. \tag{2}$$

The concept of an electrostatic potential can be used at least in a narrow region near the wall.

For insulating (multisection) walls, the condition $\delta j_\parallel = 0$ leads to [see (2)]

$\Omega = 2\pi\mu v_A/L$, $\mu = 1, 2, \ldots$. This result corresponds to ordinary standing Alfvén waves.

In the case of conducting ends, we should express $\delta j_\parallel$ in terms of the perturbation of the plasma properties near the wall. It can be shown that if electron–electron collisions are sufficiently frequent, and if the end conductivity is high, we have

$$\delta j_\parallel = \frac{\epsilon e n v_{Ti}}{2\sqrt{\pi}}\left[\frac{e\delta\varphi}{T_e} - \left(\Lambda + \frac{1}{2}\right)\frac{\delta T_e}{T_e}\right], \tag{3}$$

where $\Lambda = \ln(v_{Te}/\epsilon v_{Ti})$. Under the conditions prevailing in open systems, we would usually have $\Lambda = 5$–7. From (2) and (3) we find the dispersion relation

$$\frac{2v_A\Omega}{L}\,\mathrm{tg}\,\frac{\Omega L}{2v_A} + \frac{i\nu\Omega}{w^2} + \frac{i\Gamma^2}{w} = 0, \tag{4}$$

where

$$\nu = \frac{\omega_{Bi}^2 m_i a^2}{\tau T_e};\quad \Gamma^2 = \frac{\omega_{Bi}}{\tau}\left(\Lambda + \frac{1}{2}\right);\quad w = ka, \tag{5}$$

ω_{Bi} is the ion cyclotron frequency, and $\tau = \sqrt{\pi}L/(\epsilon v_{Ti})$ is the plasma lifetime.

For open systems (e.g., a gasdynamic system or a tandem mirror), our model of a uniform magnetic field simulates the long central solenoid. The end structures (e.g., the mirror and the expander in a gasdynamic system) are usually very short and do not contribute to the inertia of the force tube. Their role is one of determining the potential drop between the plasma of the central solenoid and the wall. In this sense they simulate the Debye sheath in our model. For a gasdynamic system the collision rate would typically be relatively high, and our results could be applied essentially unchanged. We should also replace the coefficient ϵ by the reciprocal mirror ratio $1/R$. In tandem mirrors with infrequent collisions, expression (3) might require modification (cf. Ref. 5).

In the limit of zero β (which corresponds formally to $v_A \to \infty$), the perturbation becomes a z-dependent "flute" [see (1)], and the dispersion relation becomes a quadratic equation in Ω (with a form similar to that which arises in the theory of the drift-dissipative instability). This limit could in principle also be found from the equations of Kadomtsev's paper.[2]

The instability is affected only weakly by transverse gradients of the ion density and temperature; effects of the finite ion Larmor radius (which must be taken into account if there are variations in n and T_i) lead to only some decrease in the maximum growth rate.

In the limit $\beta \to 0$ ($v_A \to \infty$) the growth rate goes through a maximum as a function of the parameter w at $w = 1.8(\nu/\Gamma)^{2/3}$. The size of this maximum is

$$(\mathrm{Im}\,\Omega)_{max} = 0{,}25\left(\frac{v_{Ti}}{L}\right)\left(\epsilon\frac{T_e}{T_i}\right)^{1/3}\left[\frac{L(\Lambda + \frac{1}{2})}{a}\right]^{2/3}.$$

Substituting in some parameter values typical of experiments in a gasdynamic system[6]

($\epsilon = 1/R = 1/20$, $T_e/T_i = 1$, $L/a = 100$; $\Lambda = 5$) we find $(\mathrm{Im}\Omega)_{\max} = 6.2(v_{Ti}/L)$.

This large growth rate makes it important to deal with the finite magnitude of the Alfvén wave velocity. It can be shown that this effect leads to a decrease in the maximum growth rate. This maximum rate cannot be greater than (or on the order of) v_A/L, in any case. This decrease in growth rate occurs even at very small values of β. Assuming $\beta = 0.2$, and otherwise retaining the numerical values used above for the parameters, we find from (4) that $(\mathrm{Im}\Omega)_{\max}$ decreases by a factor of nearly two (while remaining very large).

In summary, the joint effects of two factors—conducting ends and a transverse gradient of the electron temperature—cause a specific instability, which should lead to an equalization of the electron temperature over the plasma cross section.

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