Magnetoabsorption at quantum points

A. O. Govorov and A. V. Chaplik

Institute of Semiconductors Physics, Siberian Branch of the Academy of Sciences of the USSR

(Submitted 7 May 1990)

Pis'ma Zh. Eksp. Teor. Fiz. **52**, No. 1, 681–683 (10 July 1990)

An exact solution is derived for the problem of the frequencies and intensities of the optical absorption lines of a system of magnetized 2D electrons which are confined at a quantum point by a parabolic potential. No restriction is imposed on the interaction between particles.

The Kohn theorem[1] asserts that a resonance occurs in the absorption of a uniform rf electric field by a system of interacting electrons in a magnetic field at the cyclotron frequency, regardless of the nature of the interaction, provided that it depends on only the difference between the coordinates of particles. That result was derived for spatially uniform systems. Brey *et al.*,[2] recently showed that a similar exclusion of the interaction occurs for particles which are moving in a 1D parabolic potential. The parabolic approximation of the lateral potential of 1D microstructures (quantum wires) and zero-dimensional microstructures (quantum points) based on 2D systems has been confirmed well by theory[3] and experiments.[4,5] In the present letter we examine a quantum point in a magnetic field which is directed perpendicular to the plane of the heterojunction. The confining potential is $V(\vec{\rho}) = m\Omega^2\rho^2/2$, where m is the effective mass, and $\rho = \sqrt{x^2+y^2}$ is the in-plane distance from the center of the quantum point. We will use a method which reveals not only the exact resonant frequencies of an N-particle system in the potential $V(\vec{\rho})$ in the presence of a magnetic field but also the intensities of the corresponding lines (these intensities were not found in Refs. 1 and 2). The distinctive feature of the dynamics in a parabolic potential which makes the frequencies of optical transitions independent of the distance between electrons becomes clear in the process.

We write the Hamiltonian of our system in the symmetric gauge of the vector potential $\mathbf{A} = (1/2)[\mathbf{B}\vec{\rho}]$ in the following form ($\hbar = 1$):

$$\hat{\mathcal{H}} = -\frac{1}{2m}\left\{\sum_{k=1}^{N}\left[\partial^2/\partial\vec{\rho}_k^{\,2} + \frac{ieB}{c}\left(x_k\frac{\partial}{\partial y_k} - y_k\frac{\partial}{\partial x_k}\right) - \left(\frac{eB}{2c}\right)^2\rho_k^2\right]\right\}$$

$$+\frac{m\Omega^2}{2}\sum_R \rho_k^2 + \sum_{j<k} u(\vec{\rho}_j - \vec{\rho}_k) + \hat{H}_{spin}, \qquad (1)$$

where B is the magnetic field, $u(\vec{\rho}_j - \vec{\rho}_k)$ is the binary interaction potential and $\hat{H}_{\rm spin}$ is the spin part of the energy of the system in a magnetic field.

We replace $\vec{\rho}_k$ ($k = 1, 2, \ldots N$) by the variables $\mathbf{R}$ and $\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_{N-1}$, which make it possible to separate out the center-of-mass motion. These variables are normalized in a special way:[6]

$$\mathbf{R} = \sum_{k=1}^{N} \vec{\rho}_k/\sqrt{N}, \quad \mathbf{x}_1 = \frac{\vec{\rho}_1 - \vec{\rho}_2}{\sqrt{1\cdot 2}}, \quad \mathbf{x}_2 = \frac{\vec{\rho}_1 + \vec{\rho}_2 - 2\vec{\rho}_3}{\sqrt{2\cdot 3}},$$

$$\mathbf{x}_{N-1} = \frac{\vec{\rho}_1 + \vec{\rho}_2 + \ldots - (N-1)\vec{\rho}_N}{\sqrt{(N-1)N}}. \tag{2}$$

Transformation (2) conserves the part of Hamiltonian $\hat{\mathscr{H}}$ which does not contain the interaction $\Sigma u(\rho_j - \rho_k)$. In this term itself there is no R dependence after the transformation to the new variables. We can thus write

$$\hat{\mathcal{H}}(\mathbf{R}, \mathbf{x}_1 \ldots \mathbf{x}_{N-1}) = \mathcal{H}_0(\mathbf{R}) + \mathcal{H}'(\mathbf{x}_1 \ldots \mathbf{x}_{N-1}),$$

$$\hat{\mathcal{H}}_0 = -\frac{1}{2m}\left[\frac{\partial^2}{\partial R^2} + \frac{ieB}{c}\left(X\frac{\partial}{\partial Y} - Y\frac{\partial}{\partial X}\right)\right] + \frac{m\tilde{\omega}^2 R^2}{2}, \tag{3}$$

where the composite frequency is $\tilde{\omega} = \sqrt{\Omega^2 + \omega_c^2/4}$, $\omega_c = eB/mc$, and X and Y are components of the vector $\mathbf{R}$.

The coordinate part of the wave function of the system is thus $\Psi = \psi_{nM}(\mathbf{R})\cdot\phi(\mathbf{x}_1 \ldots \mathbf{x}_{N-1})$, where ψ_{nM} is the solution of the Schrödinger equation for a 2D isotropic oscillator in a magnetic field directed along the normal to the oscillation plane, n and M are the radial and azimuthal quantum numbers, and the energy levels are given by[7]

$$E_{nM} = (2n + |M| + 1)\tilde{\omega} + \frac{\omega_c}{2}M, \quad n = 0, 1, 2 \ldots; \; M = 0, \pm 1, \pm 2 \ldots . \tag{4}$$

The wave function $\psi_{nM}(\mathbf{R})$ is always symmetric with respect to all particles, so the Pauli principle must be satisfied by virtue of the function ϕ and the spin factor.

In the dipole approximation, the interaction with the electromagnetic field is described by the Hamiltonian $H_{\rm int} = e\mathbf{E}(t)\Sigma\vec{\rho}_k = e\mathbf{E}(t)\sqrt{N}\mathbf{R}$, which does not contain the variables $\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_{N-1}$, and which is therefore diagonal in the quantum numbers of the function ϕ. The optical absorption of this N-particle system thus looks the same as that of an isotropic 2D oscillator in the field of a wave of amplitude $\sqrt{N}E$. The resonant frequencies are $\tilde{\omega} + \omega_c/2$ and $\tilde{\omega} - \omega_c/2$ for the transitions $\Delta M = +1$ and $\Delta M = -1$, respectively. In other words, the left-hand and right-hand polarized waves are absorbed at different frequencies. The intensity of each line is proportional to the number of particles at the quantum point, N, while the magnetic field dependence of these intensities is given by the formulas (for the oscillator strengths)

$$J_+ \propto N\frac{(\tilde{\omega} + \omega_c/2)}{\tilde{\omega}}, \quad J_- \propto N\frac{(\tilde{\omega} - \omega_c/2)}{\tilde{\omega}}, \tag{5}$$

[the matrix element of the coordinate $\mathbf{R}$ between the functions $\psi_{nM}(\mathbf{R})$ is proportional to $(m\tilde{\omega})^{-1/2}$]. In a weak magnetic field $\omega_c \ll \Omega$, the intensities of the two lines are of course, equal, while at $\omega_c \gg \Omega$ we have the asymptotic behavior $J_+ \sim B^0, J_- \sim B^{-2}$.

It can be seen from the discussion above that a distinctive feature of the parabolic potential is the existence among the various natural modes of the system of a mode such that the corresponding motion does not involve the excitation of internal degrees of freedom, which are described by the coordinates $\mathbf{x}_1,...,\mathbf{x}_{N-1}$. If the interaction between particles depends on only the differences $\vec{\rho}_j - \vec{\rho}_k$, this interaction will have no effect on the given mode. Remarkably, it is this mode which is responsible for the absorption of long-wavelength electromagnetic radiation (radiation with a wavelength greater than the diameter of the quantum point). The experiments of which we are aware on IR magnetoabsorption at InSb quantum points[5] have revealed the position of the absorption peak to be independent of N, in agreement with this theory, So far, there are no results on the magnetic field dependence of the intensities. Nazin and Shikin[8] established that the frequency of a collective mode is the same as the one-particle frequency of a parabolic potential according to classical mechanics for the case of a Coulomb interaction between particles.

It is also a straightforward matter to deal with the effect of a uniform electric field in the plane of the system on a quantum point. Let us assume, for example, that the field E is directed along the y axis. The wave function then becomes

$$\Psi = e^{ip_0X}\psi_{nM}(X,Y+Y_c)\phi(\mathbf{x}_1...\mathbf{x}_{N-1}),$$

where $p_0 = -eE\sqrt{N}\omega_c/(2\Omega^2)$, and $Y_c = eE\sqrt{N}/(m\Omega^2)$. A change in the energy of this system corresponds to a common shift of all levels by an amount $\Delta E = -e^2E^2N/(2m\Omega^2)$. The polarizability of a quantum point (in its plane) is thus independent of the magnetic field and is equal to N times the polarizability of a harmonic oscillator, $e^2/(m\Omega^2)$.

[1]W. Kohn, Phys. Rev. **123**, 1242 (1961).
[2]L. Brey, N. F. Johnson, and B. I. Halperin, Phys. Rev. B **40**, 10647 (1989).
[3]S. E. Laux *et al.*, Surf. Sci. **196**, 101 (1988); A. Kumar *et al.*, Phys. Rev. B, 1990, in press.
[4]J. Alsmeier *et al.*, Phys. Rev. B **37**, 4314 (1988); F. Brinkop *et al.*, Phys. Rev. B **37**, 6457 (1988).
[5]T. Demel *et al.*, Phys. Rev. B **38**, 12732 (1988).
[6]Yu. A. Bychkov *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 152 (1981) [JETP Lett. **33**, 143 (1981)].
[7]C. G. Darwin, Proc. Cambridge Philos. Soc. **27**, 86 (1931).
[8]S. S. Nazin and V. B. Shikin, Fiz. Nizk. Temp. **15**, 227 (1989) [Sov. J. Low Temp. Phys. **15**, 127 (1989)].

Translated by D. Parsons