Fréedericksz transition induced in nematic liquid crystal by circularly polarized light wave

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It is shown theoretically that a bistability occurs, and there is a limitation on the director reorientation angle, near the threshold for the Fréedericksz transition caused in a homeotropically oriented nematic liquid crystal by a circularly polarized light wave.

Several experiments[1-3] have revealed a fundamental difference between the light-induced Fréedericksz transitions which occur in a homeotropically oriented nematic liquid crystal when the light wave is circularly polarized and linearly polarized. In Refs. 1 and 2, for example, in experiments in which self-focusing was being used as a method of study, no reorientation of the director at all was detected at power densities p three or more times the threshold $p_{\rm lin}$, for a linearly polarized wave. A subtler polarization experiment[3] revealed that the director field is distorted (in accordance with the theory of Ref. 4) at $p \geqslant 2p_{\rm lin}$, but the magnitude of this reorientation is extremely small. In addition, a bistability of the director was observed in Ref. 3. It was stated in Ref. 5 that the behavior of the liquid crystal above the threshold is complicated because of an energy exchange between extraordinary and ordinary waves caused because the director field was not planar. However, the director reorientation theory of Ref. 5, which was limited by the scope of the perturbation-theory method, and which ignored the elastic relaxation of nonplanar deformations, did not reveal the steady-state motion of the director or describe a bistability. In the present letter we offer a systematic theory for the light-induced Fréedericksz transition in a circularly polarized field.

Adopting a Cartesian coordinate system with y axis running perpendicular to the cell walls, we express the components of the director in terms of the polar angle ψ and the azimuthal angle φ:

$$n_x = \sin\psi\cos\varphi, \qquad n_y = \cos\psi \qquad n_z = \sin\psi\sin\varphi.$$

We describe the polarization state of the monochromatic plane light wave by the Stokes parameters[6] S_1, S_2, and S_3, which are defined with respect to extraordinary and ordinary waves.

The self-consistent system of equations for the optical field and the director[7] can be reduced to the form

$$\sin^2\psi\,\frac{\partial\varphi}{\partial\tau} = \frac{\partial}{\partial\eta}\left(\frac{\partial\varphi}{\partial\eta}\sin^2\psi\right) + \delta S_1\sin^2\psi\ , \qquad (1)$$

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$$\frac{\partial\psi}{\partial\tau} = \frac{\partial^2\psi}{\partial\eta^2} - \left(\frac{\partial\varphi}{\partial\eta}\right)^2 \sin\psi\cos\psi + \delta(1+S_3)\sin\psi\cos\psi \ , \tag{2}$$

$$\frac{\partial S_1}{\partial\eta} = 2N\sin^2\psi\, S_2 - 2\frac{\partial\varphi}{\partial\eta} S_3, \tag{3}$$

$$\frac{\partial S_2}{\partial\eta} = -\ 2N\sin^2\psi\, S_1 \ , \tag{4}$$

$$\frac{\partial S_3}{\partial\eta} = 2\frac{\partial\varphi}{\partial\eta} S_1 \ . \tag{5}$$

Here $\tau = t/\tau_0$ is the dimensionless time, $\eta = \pi y/L$ is the dimensionless coordinate, $\delta = p/p_0$ is the dimensionless power density, $\tau_0 = \gamma_1 L^2/\pi^2 K$, $p_0 = \pi^2 c^2 K/\Delta n L^2$, $N = \Delta n L/\lambda$, $\Delta n = \Delta\epsilon\epsilon_\perp^{1/2}/2\epsilon_\parallel$, L is the crystal thickness, γ_1 is the viscosity, $\epsilon_\perp$ and $\epsilon_\parallel$ are the principal values of the dielectric tensor, $\Delta\epsilon = \epsilon_\parallel - \epsilon_\perp$, K is the Frank elastic constant, and λ is the wavelength of the light.

Setting $\psi(\eta,\tau) = \psi_0(\tau)\sin\eta$ and $\varphi(\eta,\tau) = \varphi_0(\tau) + \varphi_1(\tau)\cos\eta$, using a perturbation theory in the parameter φ_1 to solve system (3)–(5), and using the boundary condition $S_2(\eta = 0) = 1$, we find a system of ordinary differential equations:

$$\frac{d\varphi_1}{d\tau} = -\ 3\varphi_1 + \delta f_1, \tag{6}$$

$$\frac{dQ}{d\tau} = 2Q(\delta - 1 - \frac{3}{4}\varphi_1^2 - \delta\varphi_1 f_2) - \frac{\delta Q^2}{N}, \tag{7}$$

where $Q = N\psi_0^2$,

$$f_1(Q) = \frac{8}{\pi}\int_0^\pi \sin^2\eta\cos\eta\sin\left[Q\left(\eta - \frac{\sin 2\eta}{2}\right)\right] d\eta,$$

$$f_2(Q) = \frac{4}{\pi}\int_0^\pi \sin^2\eta\, d\eta \int_0^\eta \sin\eta'\sin\left[Q\left(\eta' - \frac{\sin 2\eta'}{2}\right)\right] d\eta'.$$

The function f_1 and f_2 can be approximated quite accurately in the interval $0 < Q < 1.1$ by the simple expressions

$$f_1(Q) = -0{,}6\sin\pi Q, \qquad f_2(Q) = 1{,}1\sin(\pi Q/2).$$

The time dependence of Q and φ_1 does not depend on the angle through which the director plane rotates, φ_0 (the rate at which this plane rotates was determined in Refs. 3 and 7).

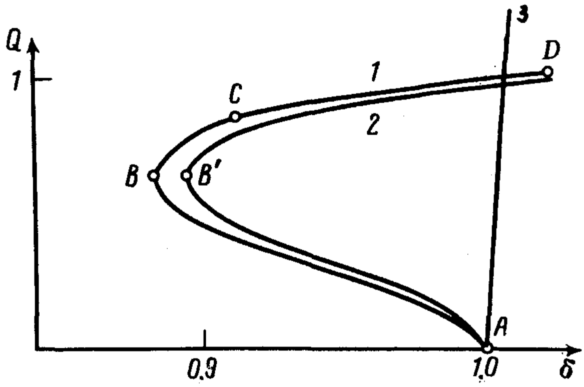

FIG. 1. *1*—Plot of $Q(\delta)$ according to (8b) at $N=\infty$; *2*—the same, for $N=18$; *3*—$Q(\delta)$ according to (10) for $N=18$.

The coordinates Q of the stationary points of system (6)–(7) satisfy the equations

$$Q = 0, \tag{8a}$$

$$\delta^2\left(f_1 f_2 + \frac{f_1^2}{4}\right) - 3\delta\left(1 - \frac{Q}{2N}\right) + 3 = 0. \tag{8b}$$

The trivial solution $Q=0$ corresponds to an unperturbed director field, and the roots of the corresponding characteristic equation are $\Gamma_1 = -3$ and $\Gamma_2 = 2(\delta - 1)$. In other words, the value $\delta = 1$ is a stability threshold. Curve *1* in Fig. 1 shows a nontrivial solution, which describes a distorted direction field (for $N=\infty$). Figure 2 shows the roots of the characteristic equation

$$\Gamma^2 + \left(3 + \frac{2}{3}\delta^2 Q f_1 \frac{df_2}{dQ}\right)\Gamma = 2\delta^2 Q \frac{d}{dQ}\left(f_1 f_2 + \frac{f_1^2}{4}\right) = 0 \tag{9}$$

as a function of Q. It follows from Figs. 1 and 2 that branch AB of the $Q(\delta)$ curve is unstable, while branch BD is stable (at point B we have $\delta_B = 0.88$ and $Q_B = 0.64$). Over an interval of width $\Delta\delta = 1 - \delta_B$, there is accordingly a bistability of the director field.

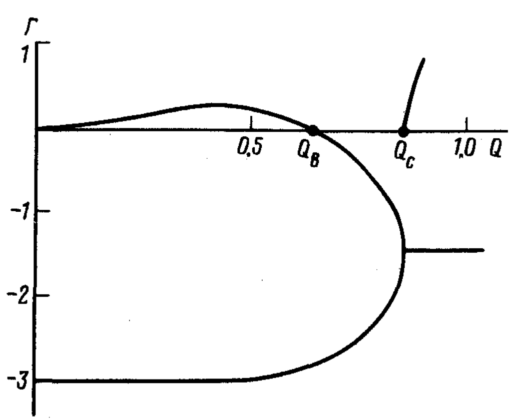

FIG. 2. Roots of characteristic equation (9) versus Q for $N=\infty$. For $Q<Q_C$, there are two real values, Γ_1 and Γ_2, while at $Q>Q_C$ the quantity Γ is complex. Lower curve—$\mathrm{Re}\Gamma$; upper curve—$|\mathrm{Im}\Gamma|$.

Let us look at some experimental results. Curve *2* in Fig. 1 shows $Q(\delta)$ according to a calculation for $N = 18$, which is the value corresponding to the experimental conditions of Ref. 3. The width of the bistability region, $\Delta\delta = 0.10$, agrees well with experimental value $\Delta\delta = 0.13$. The damped oscillations observed in Ref. 8 before the attainment of a steady-state value of Q are explained on the basis that at $Q > Q_C$ the stationary point of system (6), (7) is a stable focus.

The solution $Q(\delta)$ which we have found is fundamentally different from the linear behavior which is characteristic for Fréedericksz transitions near the threshold in linearly polarized light and in quasisteady fields. The same linear behavior,

$$Q = 2N(\delta - 1), \tag{10}$$

follows from (7) if we ignore nonplanar deformations of the director field ($\varphi_1 = 0$). Curve *3* in Fig. 1 shows a plot of (10) for $N = 18$ for comparison.

It follows from (8b) and Fig. 1 that the polar angle of the reorientation of the director, ψ, is limited to the small value $\psi_{\max} \approx N^{-1/2}$. Consequently, the light-induced Fréedericksz transition in a circularly polarized wave may not be accompanied by an aberrational self-focusing.

This theory thus reveals the director field in a circularly polarized light wave and explains the bistability and the limitation on the reorientation.

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