Cancellation of anomalies for four-dimensional heterotic strings in a Lagrangian approach

S. V. Ketov, S. M. Kuzenko, and O. A. Solov'ev
Institute of High-Current Electronics, Siberian Branch of the Academy of Sciences of the USSR

(Submitted 30 May 1990)
Pis'ma Zh. Eksp. Teor. Fiz. **52**, No. 2, 713–716 (25 July 1990)

General conditions for the cancellation of (super-) gravitational and Siegel anomalies are derived for four-dimensional heterotic strings, formulated in a two-dimensional (1,0)-supersymmetric Lagrangian field theory with the help of chiral bosons on cosets.

A covariant geometric description of four-dimensional (4D) heterotic strings and an explicit realization of the internal symmetries are possible when (1,0)-supersymmetric *chiral* bosons are used in a 2D Lagrangian field theory formulated in accordance with a curved (1,0) superspace. Other approaches to the compactification of strings, which start from the use of free fermions or covariant lattices, Calabi-Yao manifolds or orbitals, or $N=0,1,2$ minimal models or Gepner constructions,[1] are based on fundamental requirements of a superconformal invariance and a modular invariance and are not directly related to any invariant 2D action. On the other hand, minimal series can be constructed in a 2D (super)- conformal theory by the Goddard-Kent-Olive method with the help of Kac-Moody (super-) algebras, which are associated with G/H cosets. In turn, it becomes possible to find a realization within the framework of a 2D Lagrangian quantum field theory in terms of Wess-Zumino-Novikov-Witten models on G, in which models the H degrees of freedom are assigned a gauge in an appropriate way.[2] There is accordingly the possibility of constructing a corresponding covariant 2D action to describe a compactification of strings on the basis of minimal models, in particular, Gepner's construction.[1]

For 4D *heterotic* strings, such an action can be constructed in terms of (1,0)-supersymmetric chiral bosons (leftons and rightons), which are used (along with heterotic fermions) as internal degrees of freedom and which are defined on *independent* cosets G^L/H^L and G^R/H^R, respectively.[3] The general structure of an action for D-dimensional ($D\leqslant 10$) heterotic strings is[1)]

$$S=S_{HS}(X^m,\eta^I_-)+S_R(\Phi_R^{\hat{I}})+S_L(\Phi_L^{\hat{\alpha}})+S_{NM}(\Psi^{\hat{i}}_-), \tag{1}$$

where X^m ($m=0,1,...,D-1$) are scalar (1,0) superfields whose zero modes are the coordinates of a d-dimensional Minkowski space-time, η^I_- ($I=1,...,N_F$) are heterotic fermions, S_R is a chiral (1,0)-supersymmetric Wess-Zumino-Novikov-Witten action for the G^R/H^R coordinates, Φ_R^I ($\hat{I}=1,...,\dim(G^R/H^R)$) are (1,0)-supersymmetric (non-Abelian) rightons, S_L is a chiral (1,0)-supersymmetric Wess-Zumino-Novikov-Witten action for the G^L/H^L coordinates, $\Phi_I^{\hat{\alpha}}$ ($\hat{\alpha}=1,...,\dim(G^L/H^L)$) are (1,0)-supersymmetric (non-Abelian) leftons, and S_{NM} is the action of spin notons $\Psi^{\hat{i}}_-$

($\hat{i} = 1,\dots,N_\Psi$). To save space, we will omit the explicit expression for action (1).

It is important to note that the actions S_R and S_L are *chiral*, i.e., contain corresponding Lagrange multipliers in covariant derivatives which provide the required chirality for the bosons and which are invariant under associated local Siegel transformations.[4] In general, the action S_{NM} is required to cancel an accompanying Siegel anomaly (more on this below). We will use only *spinor notons*,[5] since there are no quantum obstructions associated with them, in contrast with the Hall bosons in Ref. 6 (because of limitations on the values of the Siegel Lagrange multipliers Λ: $\Lambda < 1$) and so-called B-C systems[6] (because of the existence of a dynamic state for these fields).

In quantum theory, action (1) must be supplemented with corresponding actions (in appropriate gauges) for (super-) reparametrized ghosts and for ghosts associated with Siegel symmetries.

A calculation of (super-) gravitational anomalies (by, for example, the proper-time method in the background-field formalism) makes it possible to formulate conditions for the disappearance of the net central charge in the L and R sectors in the following form:

$$\sum_i \left[\frac{\dim(G_i^L)}{1 + c_2(G_i^L)/2k_i} - \frac{\dim(H_i^L)}{1 + c_2(H_i^L)/2k_i'} + \frac{1}{2}\dim(G_i^L/H_i^L)\right] = \frac{3}{2}(10 - D),$$

$$\sum_\alpha \left[\frac{\dim(G_\alpha^R)}{1 + c_2(G_\alpha^R)/2k_\alpha} - \frac{\dim(H_\alpha^R)}{1 + c_2(H_\alpha^R)/2k_\alpha'}\right] + \frac{1}{2}N_F = 26 - D, \tag{2}$$

The summation here is over all simple factors in $G_R^L/H^L(G^R/H^R)$: $G^L/H^L = \Pi_i G_i^L/H_i^L(G^R/H^R = \Pi_\alpha G_\alpha^R/H_\alpha^R)$, and $c_2(G)$ is an eigenvalue of the second-order Casimir operator in the associated representation of G (with the generators $t^{\hat{I}}$ and the structure constants f^{IJk}). Here

$$[t^{\hat{I}}, t^{\hat{J}}] = i f^{\hat{I}\hat{J}\hat{K}} t^{\hat{K}}, \quad f_{\hat{I}\hat{J}\hat{K}} f_{\hat{N}}^{\hat{J}\hat{K}} = c_2 \delta_{\hat{I}\hat{N}},$$
$$\mathrm{tr}(t^{\hat{I}} t^{\hat{J}}) = 2k\delta^{\hat{I}\hat{J}}, \tag{3}$$

and k is the level in the Kac-Moody algebra $\hat{G}$ which is used in the Goddard-Kent-Olive construction of the representations of the (super-) conformal algebra. There are corresponding definitions for the H subgroup in G (the level k' is determined by the method by which $\hat{H}$ is injected into $\hat{G}$). This conformal model can actually be associated with several different Goddard-Kent-Olive constructions, so the choice of geometric space is also ambiguous.

Equations (2) must be supplemented with the conditions for the disappearance of the resultant Siegel anomalies separately for the L and R sectors (we use one Siegel Lagrange multiplier in each sector). They are also calculated by the proper-time method in the background-field formalism, by analogy with supergravitational anomalies.

The equations for the disappearance of the resultant coefficients of the Siegel anomalies are

$$\frac{1}{2}N_\Psi + \sum_i \left[\frac{\dim(G_i^L)}{1+c_2(G_i^L)/2k_i} - \frac{\dim(H_i^L)}{1+c_2(H_i^L)/2k_i'} \right] = 26,$$

$$\sum_\alpha \left[\frac{\dim(G_\alpha^R)}{1+c_2(G_\alpha^R)/2k_\alpha} - \frac{\dim(H_\alpha^R)}{1+c_2(H_\alpha^R)/2k_\alpha'} + \frac{1}{2}\dim(G_\alpha^R/H_\alpha^R)\right] = 15. \tag{4}$$

Equations (2) and (4) could be derived in both (1,0) superfields and in components. From the component point of view, (1,0)-*superlefton* fermions (which are dynamic) do not interact with the auxiliary Siegel fields (Lagrange multiplier), so they do not contribute to a Siegel anomaly, while the (1,0)-*superrighton* fermions (which are actually notons) *interact* with corresponding Siegel Lagrange multipliers and thus contribute to a Siegel anomaly.

Equations (2) and (4) constitute the basic results. We turn now to a discussion of several possible solutions of these equations. The first term in Eq. (4), we might note, determines only the number of notions, N_Ψ, and is thus unimportant for applications.

a) The standard ten-dimensional heterotic string corresponds to the absence of leftons and rightons, so we can restrict the discussion to Eqs. (2) with the $D=10$ and $N_F=32$ solutions, which correspond (we are invoking the condition of modular invariance) to the gauge groups spin(32)/Z_2 and $E_8\times E_8'$.

b) In the *Abelian* case, for leftons *without rightons*, which is a case corresponding to a toroidal compactification of a heterotic string, we easily find the system of equations

$$D+N_L=10,\quad D+\frac{1}{2}N_F=26,\quad \frac{1}{2}N_\Psi+N_L=26, \tag{5}$$

which leads in the 4D case ($D=4$: $N_L=6, N_F=44, N_\Psi=40$) to $SO(44)$ or $E_8\times E_8'\times E_6$ heterotic strings with an $N=4$ space-time supersymmetry and $G^L=[U(1)]^6$.

c) In the *Abelian* case, for leftons with rightons, which also corresponds to a toroidal compactification of a heterotic string, we have

$$D+N_L=10,\quad N_R+D+\frac{1}{2}N_F=26,\quad N_L+\frac{1}{2}N_\Psi=26,\quad N_R=10, \tag{6}$$

which leads in the 4D case ($D=4$: $N_L=6$, $N_R=10$, $N_F=24$, $N_\Psi=40$) to $SO(24)$ or $E_6\times E_6'$ heterotic strings with an $N=4$ space-time supersymmetry.

d) In the non-Abelian case *without leftons*,[2)] which corresponds to a compactification of ten-dimensional heterotic strings on orbifolds, we have two simple equations for $k=1$:

$$\text{rank}\,(G^R/H^R) + \frac{1}{2}\dim(G^R/H^R) = 15, \quad \frac{1}{2}N_F + \text{rank}(G_R/H_R) = 16. \tag{7}$$

The first condition can be satisfied by the choice (for example) $G^R/H^R = [SU(2)]^6$, with a dimensionality of 18 and a rank of 6. The value $N_F = 20$ then determines a gauge group of rank $10 = 5 + 5$, e.g., $SO(10)\times SO(10)$. Here $SO(10)$, which is the maximal compact subgroup in E_6, can be "raised" to E_6 by a Frenkel'-Kac construction. We thus retain the possibility of an "inflation" of the orbifold into a corresponding Calabi-Yao manifold, which also corresponds to E_6 symmetry in 4D space-time, when we use a bozonization of heterotic fermions into rightons. There are of course other possibilities, e.g., an $SO(20)$ gauge group based on 20 heterotic fermions.

e) In the *non-Abelian* case with leftons, to achieve an $N = 1$ space-time supersymmetry, it is natural to follow Gepner[1] and use a combination of $N = 2$ minimal models, which are realized in this case by means of (1,0)-supersymmetric cosets, $\underline{S}U(2)_k/\hat{U}(1)$. A (2,0)-supersymmetry on the world sheet is necessary and sufficient for an $N = 1$ space-time supersymmetry. In the 4D case, the first equation in (2) thus leads to the standard condition[1]

$$\sum_{i=1}^{i_0} 3k_i/(k_i + 2) = 9, \tag{8}$$

168 solutions of which are tabulated in Ref. 7. For modular invariance, the (2,0)-supersymmetry must be expanded to a (2,2)-supersymmetry. This expansion is essentially achieved through the use of the same structural G/H blocks in the R sector as were used in the L sector.[1] In this case the number of possibilities increases only to 1176, when we allow for the choice of various modular-invariant partition functions in accordance with the A–D–E classification.[7]

"Borrowing" 9 units of the central charge in the second equation in (4), we find the following condition for the remainder $(G^R/H^R)'$ after (G^L/H^L) is inserted into (G^R/H^R), $(G^R/H^R) = (G^L/H^L)\otimes(G^R/H^R)'$ with $k = 1$:

$$\text{rank}(G^R/H^R)' + \frac{1}{2}\dim(G^R/H^R)' = 6, \tag{9}$$

where

$$N_F = 14 + \frac{1}{2}\dim(G^R/H^R). \tag{10}$$

For example, condition (9) could be satisfied for $(G^R/H^R)' = SU(3)$ with a dimensionality of 8 and a rank of 2. We would then have $N_F = 22 + 2i_0$. In particular, for the 3^5 model of Ref. 1 $(5\cdot 9/5 = 9)$ we have $N_F = 32$.

All the cases discussed above are exactly solvable through the use of a toroidal compactification or minimal models, while possibilities d) and e) may be useful for constructing realistic models of 4D heterotic strings and for phenomenological applications.

We are indebted to D. Gepner and A. Yu. Morozov for useful discussions.

[1)] Our notation and conventions are the same as those of Ref. 3.
[2)] In this case there are no notons ($N_\Psi = 0$), since the first equation in (4) is not involved.

---

[1] D. Gepner, Phys. Lett. B **199**, 380 (1987); Nucl. Phys. B **296**, 757 (1988).
[2] D. Karabali, Q.-H. Park, and H. J. Schnitzer, Phys. Lett. B **216**, 307 (1989).
[3] S. J. Gates and W. Siegel, Phys. Lett. B **206**, 631 (1988).
[4] W. Siegel, Nucl. Phys. B **238**, 307 (1984).
[5] S. M. Kuzenko and O. A. Soloviev, Int. J. Mod. Phys. A **5**, 1341 (1990).
[6] C. M. Hull, Phys. Lett. B **206**, **212**, 419.
[7] M. Lynker and R. Schimmrigk, Phys. Lett. B **208**, 216 (1988); **215**, 681.

Translated by D. Parsons