Entropy of the choice of vacuum during phase transitions

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An asymptotic expansion of the logarithm of the partition function, $\ln Z$, in the number of lattice sites, N, which is allowed to go to infinity, is discussed. There is a jump in the zeroth-power term at the transition point. For the models on lattices with the topology of a torus which have been studied, the magnitude of the jump turns out to be $\ln Q$, where Q is the order of the broken symmetry group.

Among the many interesting properties of phase transitions, there is one which is common to most transitions, regardless of their order. This is the property that the system itself chooses a vacuum as it passes through the transition point.

Our purpose in the present study was to seek manifestations of this property.

We recall that a phase transition appears only in the limit of an infinite lattice volume. We accordingly consider an asymptotic expression for the free energy (in temperature units):

$$\ln Z = f_0(B) \ + \ \ldots f_1(B) \ + O(1). \tag{1}$$

In principle, depending on the boundary conditions, there could be other terms in (1), with integer powers, $N^{1/d}$; there could also be terms proportional to $\ln N$ (the latter possibility corresponds to the point of a second-order transition on a solid lattice or for models on a random lattice).

We are ordinarily interested in the behavior $f_0(B)$. This is a continuous function with respect to B, while its derivatives (depending on the order of the transition) are singular.

If we examine the Ising model on a torus, which has been solved by Onsager (for a finite lattice),[1] we find

$$\lim_{\epsilon_1 \to 0} f_1(B_c + \epsilon_1) - \lim_{\epsilon_2 \to 0} f_1(B_c - \epsilon_2) = \ln 2. \quad (2)$$

This result agrees with the simplest expression which could be expected. If a symmetry group with order Q_2 is broken to a subgroup with order Q_1, then we could reasonably expect

$$\Delta f_1(B) = \ln Q_2/Q_1. \quad (3)$$

The numerical method of Sogo and Kimura[2] has been used to study Potts models with $Q=3$ and $Q=5$ on 2D lattices with dimensions up to 20×20.

Within the error of our numerical simulation ($\sim 30\%$ after 100 000 iterations), the value of the discontinuity, $\Delta f_1(B_c)$, agreed with ln3 and ln5, respectively (for second-order and first-order transitions).

A corresponding result, ln2, was found for a $d=3$ Ising model (with a lattice size up to 7^3).

We used the results of Kazakov[3] to analytically calculate values of $f_1(B)$ for an Ising model on a random lattice with the topology of a sphere or a torus. These are third-order transitions. In the case of the topology of a sphere, the effect unexpectedly disappeared:

$$\lim_{\epsilon \to 0} f_1(B_c + \epsilon) - f_1(B_c - \epsilon) = 0. \quad (4)$$

In the case of the torus, it reappeared:

$$\lim_{\epsilon \to 0} f_1(B_c + \epsilon) - f_1(B_c - \epsilon) = \ln 2. \quad (5)$$

Calculations of $f_1(B)$ are presently being carried out for Ising models on solid lattices with the topology of a sphere or a pretzel (these are numerical calculations) and on random lattices of types $g=2$ and 3.

It would be interesting to study the universality question and how the effect depends on lattice defects. A corresponding effect might be expected in stochastic phenomena. At any rate, if, in a doubling of the period of cycles, we define Z as the maximum inflation of the original differential uncertainty, then we find that the jump again agrees with (3).

Finally, this effect might turn out to be important in the case of the breaking of a local symmetry group. In such a situation (spin glasses?), we might find the result $\ln Q_2/Q_1 \sim N$.

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