Bistability of above-threshold oscillations of parametric magnetoelastic waves in hematite

A. O. Kiselev, V. I. Ozhogin,[1)] and V. L. Preobrazhenskiĭ
Moscow Institute of Radio Engineering, Electronics, and Automation

(Submitted 1 June 1990)
Pis'ma Zh. Eksp. Teor. Fiz. **52**, No. 2, 726–729 (25 July 1990)

The results of an experimental observation of a double parametric resonance of quasiacoustic waves in an α-Fe_2O_3 crystal are reported. A bistability has been observed in the oscillations of the intensity of the parametric quasisound. This bistability results from an effective anharmonicity of mixed magnetoelastic modes.

Crystals of high-temperature antiferromagnets with an easy-plane anisotropy, e.g., α-Fe_2O_3 and $FeBO_3$, are distinguished by an extremely high sensitivity of their sound velocities to changes in the magnetic field and by a giant effective elastic anharmonicity.[1] These effects are seen clearly in experiments on the parametric excitation of quasiacoustic magnetoelastic waves by a longitudinal magnetic pump.[2] In the present letter we are reporting an experimental observation of a resonant excitation and a bistability of oscillations in the above-threshold intensity of quasisound in the case of a composite pump, i.e., a rapidly varying pump plus a slowly varying pump.

In the experiments we used a thin-disk sample of a α-Fe_2O_3 crystal with a diameter of 5 mm and a thickness of 0.35 mm, cut in the basal plane. The static magnetizing field **H** was directed parallel to the basal plane. A rapidly varying field of a parametric pump, $\mathbf{h}_p(t) = h_p \cos \omega_p t$, and also a slowly varying ("modulating") field $\mathbf{h}_M(t) = h_M \cos \omega_M t$ ($\mathbf{h}_p \| \mathbf{h}_M \| \mathbf{H}$, $\omega_p \gg \omega_M$) were applied to the sample by means of an inductance coil. A pickup coil oriented perpendicular to the exciting coil detected oscillations of the magnetization in the basal plane accompanying the varying elastic strain. The experimental geometry is shown in the inset in Fig. 2. When the amplitude (h_p) of the pump field, of frequency $\omega_p/2\pi \approx 1$ MHz, exceeds a threshold, $h_p > h_{pc}$, a parametric instability of a quasiacoustic contour-shear mode[3] with a frequency $\omega_n/2\pi \sim 0.5$ MHz occurs in the sample. Figure 1, which is a photograph of the screen of a cathode-ray curve tracer, shows the typical behavior of the intensity of the parametric oscillations as a function of the pump frequency at $H = 0.7$ kOe and $h_p/h_{pc} \approx 5$. When a modulating field with a certain frequency $\omega_M \approx 2\Omega_M$, where $\Omega_M \approx 1$ kHz, is imposed, we observe a resonant excitation of oscillations in the intensity of the quasisound (the thickening of the trace in Fig. 1). The excitation is of a parametric nature, and it involves a threshold in the amplitude of the modulating field, h_M. The observed effects are acoustic analogs of double parametric resonance processes which were studied previously during the excitation of collective oscillations of parametric magnons in ferrites and antiferromagnets.[4,5] The oscillations frequency Ω_M is determined by the properties of the steady (in the absence of modulation) above-threshold state of the parametric quasisound. Figure 2 shows a typical experimental plot of the parametric-resonance frequency ($\omega_M = 2\Omega_M$) versus the frequency deviation ($\Delta\omega_n = \omega_p - 2\omega_n$)

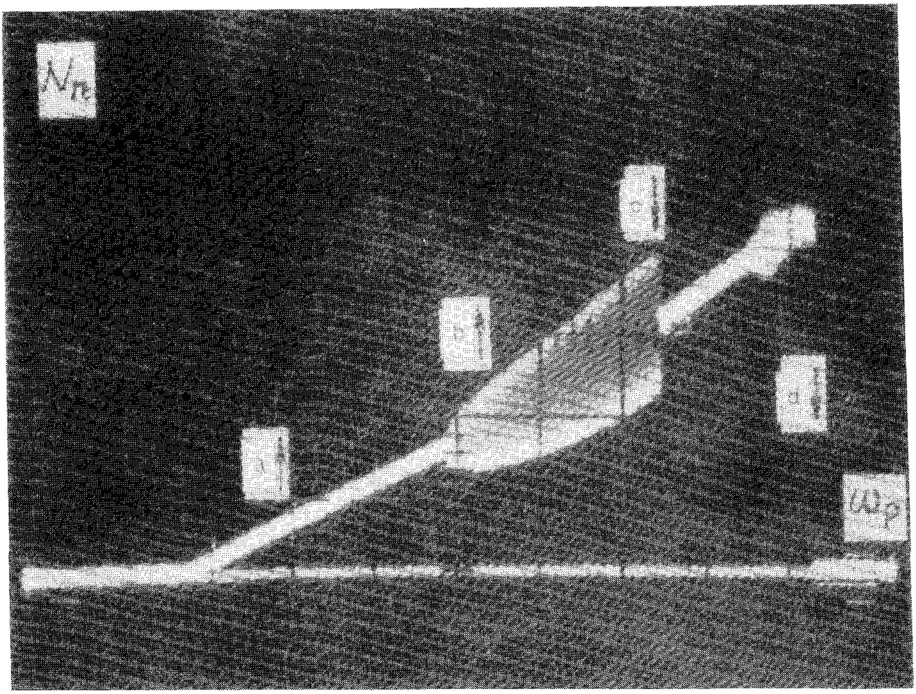

FIG. 1. Intensity of the parametric oscillations versus the pump frequency ω_p. The thickening of the trace corresponds to oscillations in the intensity. *a*—Frequency at which the parametric instability appears as ω_p is reduced; *b*—the same, for parametrically excited oscillations in the quasisound intensity; *c,d*—frequencies at which the oscillations of the intensity and the parametric generation of quasisound, respectively, are cut off as ω_p is increased.

of the rf coil. A distinctive feature of the parametrically excited quasisound intensity oscillations is their bistability, which is clearly expressed in a hysteresis in the plot of the oscillation amplitude versus the modulation frequency ω_{M} (Fig. 3).

The behavior observed can be explained by the theory of the parametric excitation of quasiacoustic modes incorporating an effective fourth-order elastic anharmonicity, which determines the nonlinear frequency shift of the magnetoelastic mode.[6,1] The complex amplitude of the parametric quasisound, b_n, is described by the equation[2]

$$\dot{b}_n + \frac{1}{2}\omega_n Q_n^{-1} b_n + \frac{i}{2}\Delta\omega_n b_n - i\psi_n |b_n|^2 b_n = i h_p V_n b_n^*, \tag{1}$$

where $V_n \equiv \frac{1}{2}(\partial\omega_n/\partial H)$ is the amplitude of the parametric coupling, ψ_n is the amplitude of the four-quasiphonon interaction, and Q_n is the quality factor of the mode. Corresponding to the above-threshold steady state is the normalized sound intensity $N_n \equiv 2\psi_n |b_n|^2/\omega_n$, which is given by

$$N_n = \frac{\Delta\omega_n}{\omega_n} + Q_n^{-1}\sqrt{\left(\frac{h_p}{h_{pc}}\right)^2 - 1}. \tag{2}$$

The linear dependence of the intensity of the frequency deviation agrees with the experimental results (Fig. 1). The oscillation frequency of small perturbations of the steady state is determined from Eq. (1):

$$\Omega_M = \omega_n [N_n (N_n - \frac{\Delta\omega_n}{\omega_n}) - \frac{1}{4} Q_n^{-2}]^{1/2}. \tag{3}$$

The line in Fig. 2 shows the result of a calculation of $\Omega_M^2(\Delta\omega_n) = \omega_M^2/4$ for the parameter values corresponding to the experimental conditions ($Q_n^{-1} = 2\times 10^{-4}$, $h_p/h_{pc} = 5$). It follows from (1) that the phase of the parametric quasisound, $\theta_n = 2\arg(b_n)$ is given by

$$\ddot{\theta}_n + \omega_n Q_n^{-1}\dot{Q}_n + (\omega_n Q_n^{-1} - 2h_p V_n \sin\theta_n)(\Delta\omega_n - 2h_p V_n \cos\theta_n) = 0, \tag{4}$$

where the frequency deviation is $\Delta\omega_n = \omega_p - 2\omega_n - 2(\partial\omega_n/\partial H)h_M \cos\omega_M t$, when the modulation of the quasisound spectrum is slow. Taking the nonlinearity of Eq. (4) into account within terms cubic in the deviations of the phase from its steady-state value, $\chi = \theta_n - \theta_n^0 = d\exp[i(\omega_M/2)t] + \text{c.c.}$, we find an equation like (1) for the

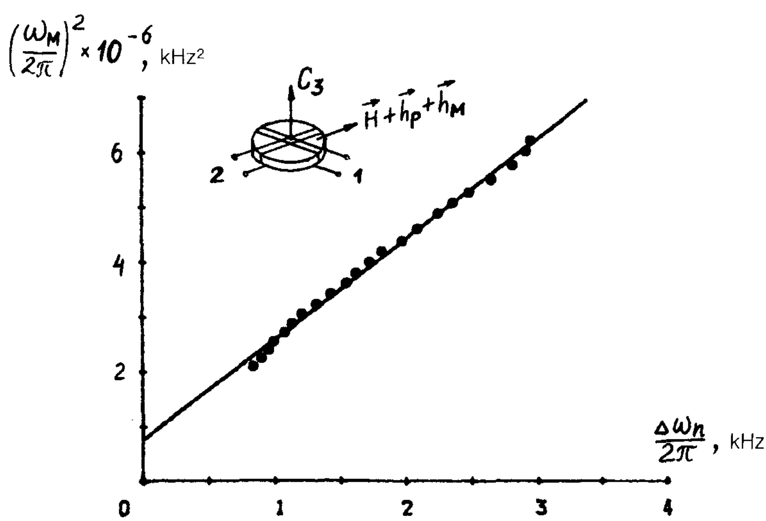

FIG. 2. Frequency of the parametric resonance of above-threshold intensity oscillations versus the frequency shift of the rf pump. The line is theoretical. In the inset, *1* and *2* are the exciting and pickup coils, and C_3 is the threefold crystallographic axis.

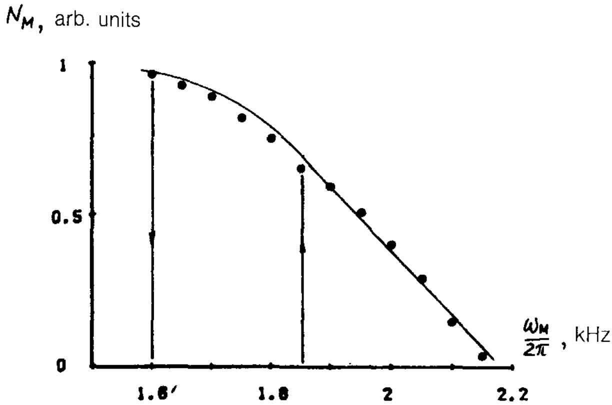

FIG. 3. Square of the amplitude of the above-threshold oscillations in the quasisound intensity versus the modulation frequency.

complex amplitudes d of the parametric oscillations of the phase. The frequency shift, the relaxation rate, the threshold modulation field, the amplitude of the parametric coupling, and the amplitude of the nonlinear self-effect are, respectively, $\Delta\omega_M = \omega_M - 2\Omega_M$, $\frac{1}{2}\Omega_M Q_M^{-1} = \frac{1}{2}\omega_n Q_n^{-1}$,

$$h_{Mc} = \Omega_M \left[\frac{\partial\omega_n}{\partial H}\sqrt{\left(\frac{h_p}{h_{pc}}\right)^2 - 1}\right]^{-1},$$

$$V_M = -\frac{\partial\omega_n}{\partial H}\left(\frac{\omega_n}{2\Omega_M}\right) Q_n^{-1}\sqrt{\left(\frac{h_c}{h_{pc}}\right)^2 - 1},$$

$$\psi_M = -\frac{\Omega_M}{N_n}\left[\frac{1}{4}\frac{\Delta\omega_n}{\omega_n} + Q_n^{-1}\sqrt{\left(\frac{h_p}{h_{pc}}\right)^2 - 1}\right].$$

In agreement with the experimental results (Fig. 3), the nonlinear frequency shift of the oscillations is negative ($\psi_M < 0$). This result is opposite that for the nonlinear frequency shift of the original quasisound ($\psi_n > 0$). Corresponding to the frequency shifts

$$\Delta\omega_M > \Omega_M Q_M^{-1}\sqrt{\left(\frac{h_M}{h_{Mc}}\right)^2 - 1} \equiv \Delta_M$$

are metastable excited states. After the oscillations are cut off, they can be restored by increasing the pump frequency ω_M in the region of the absolute instability, $|\Delta\omega_M| < \Delta_M$. This effect explains the hysteresis in the oscillation amplitude observed experimentally.

The experiments show (Fig. 3) that at low excitation levels the square of the oscillation amplitude N_M is a linear function of the frequency shift $\Delta\omega_M$. To describe this behavior, it is sufficient to retain the cubic nonlinearity in Eq. (4). At higher excitation levels we observe a significant deviation from linearity; we also observe a cutoff of the metastable state with increasing frequency shift $|\Delta\omega_M|$. A description of these effects will require a more comprehensive consideration of the nonlinearity of Eq. (4) and, possibly, the incorporation of a nonlinear damping of the quasiphonons.

These results show that when the double parametric resonance is applied to the nonlinear acoustics of magnetically ordered crystals, one can see some qualitatively new features of the above-threshold state of parametric quasiacoustic waves. In particular, it becomes possible to observe sequences of bistable above-threshold excitations.

In conclusion we wish to thank N. A. Ékonomov and L. M. Krutyanskiĭ for assistance in the experiments.

[1)] I. V. Kurchatov Institute of Atomic Energy, Moscow.

[1] V. I. Ozhogin and V. L. Preobrazhenskiĭ, Usp. Fiz. Nauk **155**, 593 (1988) [Sov. Phys. Usp. **331**, 713 (1988)].
[2] N. N. Evtikhiev, V. L. Preobrazhenskiĭ, M. A. Savchenko, and N. A. Ěkonomov, Vopr. radioelektron. Ser. obshchetekhn., No. 2, 124 (1978).
[3] E. A. Andrushchak, N. N. Evtikhiev, S. A. Pogozhev *et al.*, Akust. Zh. **27**, 170 (1981) [Sov. Phys. Acoust. **27**, 93 (1981)].
[4] V. V. Zautkin, V. S. L'vov, B. I. Orel, and S. S. Starobinets, Zh. Eksp. Teor. Fiz. **72**, 272 (1977) [Sov. Phys. JETP **45**, 143 (1977)].
[5] V. I. Ozhogin, A. Yu. Yakubovskiĭ, A. V. Abryutin, and S. M. Suleĭmanov, JMMM **15–18**, Part II, 757 (1980).
[6] V. L. Preobrazhenskiĭ, M. A. Savchenko, and N. A. Ěkonomov, Pis'ma Zh. Eksp. Teor. Fiz. **28**, 93 (1978) [JETP Lett. **28**, 87 (1978)].

Translated by D. Parsons