Stratification and superconducting droplets in high-T_c superconductors

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A model describing a stratification of a high-T_c superconductor into metallic and insulating phases is proposed. The onset of a superconducting order might be the reason for the stratification. It would then become possible to explain the appearance in a weak magnetic field, of a nonzero resistance in a $Ba_{1-x}K_xBiO_3$ sample which is a superconductor according to the Meissner effect. The resistance which appears is higher than the resistance of the normal phase.

Since the first experiments[1] on $La_2CuO_{4+\delta}$ samples, carried out in 1988, there have been a fair number of indications that certain high-T_c superconductors may undergo a stratification into metallic and insulating phases. Among these indications, in our opinion, are the results of some recent experiments,[2] in which a significant increase in the resistance of a sample was observed in a weak magnetic field after the transition to the superconducting phase (the existence of the superconducting phase was inferred from the Meissner effect).

Specifically, this result can be explained under the assumption that the appearance of a superconductivity in this system causes the sample to stratify into metallic (superconducting) droplets separated by insulating regions and coupled by a weak link. The destruction of this weak link by a weak magnetic field gives rise to a resistance in the superconducting phase. This resistance is higher than that in the normal metallic state (the resistance is now determined by the insulating regions which have appeared).

In the present letter we propose a model that describes a stratification which may occur in the normal phase, and which may also be induced by the onset of superconductivity.

We start from the picture of a high-T_c superconductor as a doped semiconductor. To describe the conversion of the charge-carrier spectrum to an insulating nature, we use a model with a Fermi surface which satisfies the nesting condition $\epsilon(\mathbf{k}) \approx -\epsilon(\mathbf{k}+\mathbf{Q})$. An exact nesting of the Fermi surface is known to cause an instability with respect to a transition to a state with a charge density wave or a spin density wave. These waves are described by the introduction of a uniform insulating order parameter $\Sigma \sim \langle a^+_{\mathbf{k}+\mathbf{Q}} a_{\mathbf{k}} \rangle$, which determines a gap in the spectrum of elementary excitations. For the discussion below, it is sufficient that there by a pseudogap due to the onset of a corresponding short-range order. The carriers which appear above (or below) the gap upon doping reduce the size of the gap and thereby reduce μ. The increase in μ due to the increase in the kinetic energy is slowed substantially because of the high density of states at the edge of the gap $[\rho(\epsilon) \sim 1/\sqrt{\epsilon - \Sigma}]$. As a result, the derivative of the chemical potential μ with respect to the number of particles, n, becomes negative ($\partial\mu/\partial n < 0$). This means an instability with respect to the appearance of macroscopic regions (an instability with $q = 0$) with an elevated particle density, and it suggests the possibility of droplet formation. In the 1D models which are amenable to exact solution,[3] this possibility does not arise, as we know, since in such models there is always an instability with respect to the formation of a superstructure with a vector $q \neq 0$, associated with a new Fermi level. The latter instability turns out to be dominant.

We will show below that this instability is weakened considerably when we incorporate both a deviation from a 1D case and commensurability effects (in the simplest case, a doubled chain potential in $YBa_2Cu_3O_7$). As a result, the formation of macroscopic metallic droplets becomes more favorable. For simplicity in all the calculations below, we use an isotropic two-band model, which is essentially equivalent to a single-band model with nesting.

Adding a term

$$H_I = \sum_{\mathbf{k}} \Sigma_I a^+_{\mathbf{k}+\mathbf{Q}} a_{\mathbf{k}} + \text{H.a.},$$

to the Hamiltonian, where $\Sigma_I = \text{const}$ serves as a source for the parameter Σ, we take the commensurate potential into account in the simplest way.

As is shown below, under the condition

$$g\frac{\Sigma^{(0)}}{\Sigma_I} \equiv n_c \ll 1$$

(g is the dimensionless coupling constant in the insulating channel, and $\Sigma^{(0)}$ is the gap in the spectrum of the undoped system), an inhomogeneous state can thus arise only if there is a slight doping ($n/\Sigma^{(0)} \ll 1$), so this stage is determined entirely by the quasiparticles of one conduction band.

The effective Hamiltonian for those quasiparticles is

$$H_{eff} = \int d\mathbf{r}\{\sum_\sigma \varphi^+_\sigma(\mathbf{r})[\frac{\xi^2(\nabla)}{2\Sigma^{(0)}} - \mu + u(\mathbf{r})]\varphi_\sigma(\mathbf{r}) + \frac{1}{2}\frac{u^2(\mathbf{r})}{n_c}\}. \quad (1)$$

Here $\varphi^+_\sigma(\mathbf{r})$ is the quasiparticle creation operator, $\xi(\mathbf{k}) = v_F(|\mathbf{k}| - k_F)$ is a seed dispersion law, and the self-consistent potential $u(\mathbf{r})$ is related to the parameter $\Sigma(\mathbf{r})$ by

$$\Sigma(\mathbf{r}) = \Sigma^{(0)} + u(\mathbf{r}), \quad u(\mathbf{r})/\Sigma^{(0)} \ll 1.$$

Working from (1), we can calculate the free energy of the homogeneous system: $f_T(n) = \langle H_{eff}\rangle + \mu n$.

At $T = 0$ we find, in dimensionless variables,

$$f_0(\eta)=\frac{1}{6}\eta^3-\frac{1}{2}n_c\eta^2;\ \mu=\frac{\partial f_0}{\partial\eta}=\frac{1}{2}\eta^2-\eta_c\eta;\ \frac{\partial\mu}{\partial\eta}=\eta-n_c, \tag{2}$$

where $\eta=n/[4N(0)\Sigma^{(0)}]$ is the dimensionless density of particles.

It can be seen from the expression for $\partial\mu/\partial\eta$ that at $\eta<n_c$ the homogeneous metallic phase is unstable, and there may be a stratification into a phase with $\eta=0$ and a relative volume of $1-v$ (an insulator) and a phase with $\eta>n_c$ and a volume v.

We write the energy of this state in the form $(T=0)$

$$F_0(\eta,v)=vf_0(\eta/v). \tag{3}$$

Minimizing F_0 with respect to v, we find an expression for the equilibrium volume v_r:

$$v_r=\frac{2}{3}\frac{\eta}{n_c}. \tag{4}$$

Under the condition $v_r\leqslant 1$ it follows from (4) that the stratified state exists at $\eta<3n_c/2\equiv\eta_0$.

It can be shown that an instability with $q\neq 0$ appears if $\eta\leqslant\eta^*\approx 1.3n_c<\eta_0$. Accordingly, as the density is reduced a stratification occurs earlier, and it is preferable from the energy standpoint to a superstructure with $q\neq 0$. In addition, the circumstance that the equilibrium density in the droplet, η_0, is higher than η^* makes inhomogeneities with any value of q unfavorable and thus makes a positive surface energy unfavorable. The latter point may lead to the formation of droplets with a macroscopic volume, as was assumed in expression (3) for the energy. The small value of the quantity $1/L$, where L is the size of a droplet, can justify ignoring the surface component in (3). In general, the size of the droplets and their spatial ordering should be determined by the Coulomb interaction associated with the local deviation from elec-

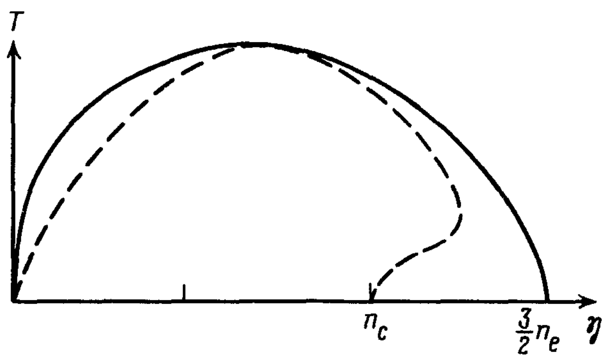

FIG. 1. (T,η) phase diagram. Solid line—Line of first-order phase transition to stratified state; dashed line—line of absolute instability of homogeneous phase $(\partial\mu/\partial n=0)$.

trical neutrality. The Coulomb interaction is suppressed to a large extent by the large dielectric constant, and it may be partially screened by a redistribution of the dopant.

Analyzing the case $T \neq 0$ in a smaller way, we find the phase diagram in Fig. 1.]

We now consider the effect of superconductivity on the formation of a stratified state. The superconductivity is described by the Hamiltonian

$$H = H_{eff} + \int d\mathbf{r}\{\Delta\varphi_{\uparrow}^{+}(\mathbf{r})\varphi_{\downarrow}^{+}(\mathbf{r}) + \text{H.a.}\}, \tag{5}$$

where

$$\Delta = -\lambda\langle\varphi_{\uparrow}(\mathbf{r})\varphi_{\downarrow}(\mathbf{r})\rangle$$

is a superconducting order parameter, and λ is the modulus of the coupling constant in the Cooper channel.

Calculating the free energy of this system for densities $\eta/(2\lambda) \ll 1$, we find, at $T = 0$,

$$f_0(\eta) \approx -\frac{\Delta^2}{4\lambda} - \frac{1}{2}n_c\eta^2. \tag{7}$$

The solution of self-consistency equation (6) in this case is

$$\Delta = \pi\lambda^{3/2}\sqrt{\eta/2}\,. \tag{8}$$

Using (8), we find the following equation from expression (7) for f_0 under the condition $\eta/(2\lambda) \ll 1$:

$$\frac{\partial\mu}{\partial\eta} = \frac{\partial^2 f_0}{\partial\eta^2} = -n_c < 0. \tag{9}$$

It follows from the expression for $\partial\mu/\partial\eta$ in (2) and (9) that under the condition $\eta_c/(2\lambda) \ll 1$ the region in which the homogeneous metallic state is unstable may expand considerably toward larger values of η. The appearance of a superconductivity in the system thus results in a loss of the stability of the homogeneous sample in the normal phase, with the result that metallic droplets weakly linked with each other form; they are separated by insulating regions. The destruction of a weak link by a magnetic field gives rise to a resistance of the sample in the superconducting phase (superconducting according to the Meissner effect). The resistance which appears is higher than that in the normal metallic state, since it is determined by the resistance of the insulating interlayers. The mechanism described here can thus be proposed as an explanation of the anomalies, mentioned above, which have been observed in the $Ba_{1-x}K_xBiO_3$ system.[2] According to the picture drawn here, the nonmonotonic temperature dependence of the critical current observed in Ref. 2 may be determined by a decrease in the carrier density in the insulating regions during cooling.

[1] J. D. Jorgensen *et al.*, Phys. Rev. B **38**, 1137 (1988).
[2] N. V. Anshukova *et al.*, Zh. Eksp. Teor. Fiz. **97**, 1635 (1990) [Sov. Phys. JETP (to be published)].

[3] S. A. Brazovskiĭ, S. A. Gordyunin, and N. N. Kirova, Pis'ma Zh. Eksp. Teor. Fiz. **31**, 486 (1980) [JETP Lett. **31**, 456 (1980)].

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