Precritical amplification of collision integral in heavy-ion reactions

L. Münchow
Joint Institute for Nuclear Research

(Submitted 5 June 1990)
Pis'ma Zh. Eksp. Teor. Fiz. **52**, No. 3, 773–775 (10 August 1990)

The interaction of particles with density fluctuations becomes the governing dissipative process near a phase instability.

Nuclear reactions involving heavy ions over the energy range from 20 to 100 MeV/A are frequently described kinetically, and the physical motivation is formulated in a thermodynamic way: to learn about the equation of state of the nuclear matter over a wide region on the P, ρ diagram. In particular, an analog of a liquid-gas phase transition is expected to appear (Refs. 1 and 2, for example). This transition is of considerable interest, particularly in connection with astrophysical models for supernova explosions.[1]

Experimentally, there are certain indications of a phase transition. In the first place, the process of multifragmentation can serve as a criterion on the onset of an instability.[3] Recent experiments[4] show a transition from a sequential emission of clusters to an instantaneous emission in a collision of Ar with V at an energy of 75 MeV/A. At the same energy, the transverse peak disappears, indicating that the effective compressibility $\varkappa$ of the nuclear matter is small. By virtue of the relation $(\partial P/\partial\rho)_T \propto \varkappa \to 0$, the disappearance of this peak has the qualitative meaning that we are close to the spinodal region. Finally, the emission is observed to be substantially isotropic.[4] This isotropy implies an intensification of thermalization processes. The relaxation time is estimated from[5] $\tau_N = T^{-2} + 0.235\rho_0/(\rho T^{-1/2})$ $[10^{-21}$ s]. According to model-based calculations,[6] the maximum temperature is $T = 5$ MeV and is nearly independent of the excitation energy. We thus have $\tau_N \sim 10^{-22}$ s, while the minimum relaxation time of the nuclear system is[7] 0.5×10^{-22} s. It is thus worthwhile to seek additional relaxation processes.

In this letter we wish to call attention to the circumstance that collisions of particles with precritical density fluctuations become the governing dissipative mechanism as $(\partial P/\partial\rho)_T \to 0$, and the corresponding relaxation time satisfies $\tau_F < \tau_N$. For a quantitative description we use the Boltzmann equation in the form[8,9]

$$\left(\frac{\partial}{\partial T} + \frac{p}{m}\frac{\partial}{\partial R}\right) f(\mathbf{p}; \mathbf{R}, T) = I[f] \equiv -i\Sigma^{<}(\mathbf{p}, \epsilon(\mathbf{p}); \mathbf{R}, T)[1 - f(\mathbf{p}; \mathbf{R}, T)]$$
$$- i\Sigma^{>}(\mathbf{p}, \epsilon(\mathbf{p}); \mathbf{R}, T) \times$$
$$\times f(\mathbf{p}; \mathbf{R}, T), \qquad \epsilon(\mathbf{p}) = \mathbf{p}^2/2m, \tag{1}$$

where $f(p,\mathbf{R},T)$ is the Wigner function, and parts of the mass operator $\Sigma^{\lessgtr}(p,\omega;R,T)$ are given by

$$-i\Sigma^{>}(11')=\langle j^{+}(1)j(1')\rangle, \qquad i\Sigma^{<}(11')=\langle j(1')j^{+}(1)\rangle,$$
$$j(1)\equiv j(\mathbf{x}_1t_1)=\int d\mathbf{x}_2 V(\mathbf{x}_1-\mathbf{x}_2)\psi^{+}(\mathbf{x}_2t_1)\psi(\mathbf{x}_2t_1)\Psi(1). \tag{2}$$

Here we have replaced the coordinates and times $\mathbf{x}_1$, t_1 and $\mathbf{x}_2$, t_2 by $\mathbf{r}=\mathbf{x}_1-\mathbf{x}_2$, $t=t_1-t_2$, $\mathbf{R}=(\mathbf{x}_1+\mathbf{x}_2)/2$, $T=(t_1+t_2)/2$, and we have then taken Fourier transforms in the variables r and t. In this manner we determine Green's function $G^{\lessgtr}(11')$, which we use to calculate the distribution functions $f(\mathbf{p};\mathbf{R},T)$.

To take the density correlation into account, we need to go to the two-particle level, introducing (for example) a T matrix in the particle-hole channel or a density correlation function. In T-matrix terms

$$\Sigma^{\lessgtr}(\mathbf{p},\omega;\mathbf{R},T)=\int\frac{d\mathbf{p}_1}{(2\pi)^3}\frac{d\omega_1}{2\pi}\text{ and }G^{\lessgtr}(\mathbf{p}_1,\omega,\mathbf{R},T)T^{\lessgtr}(\mathbf{p},\mathbf{p}_1,\mathbf{p}_1-\mathbf{p};\omega-\omega_1) \tag{3}$$

we need to determine the part of the collision term which stems from the scattering of particles by density fluctuations. As in the steady state, the T matrix has, as a function of ω, poles at frequencies of collective density oscillations ω_c. The residues at these poles are $\sim\omega_c^{-1}$. Near the instability point, certain modes become soft; i.e., $\omega_c\to 0$. As a result, this mechanism leads to an increase in the collision term in the precritical region.

We now note that $\Gamma=i\Sigma^{>}-i\Sigma^{<}$ is the width of a single-particle state. It is not difficult to see[9] that $\tau=\hbar/\Gamma$ is a relaxation time, so the time scale for the relaxation to equilibrium decreases near the critical point. This effect might increase the isotropy.

Interestingly, there is a simple relationship between the width $\Gamma(\mathbf{p},\omega;\mathbf{R},T)$ and the density correlation function $F^{>}(12)=\langle\rho(1)\rho(2)-\rho^2\rangle$, $F(12)=\langle\rho(2)\rho(1)-\rho^2\rangle$. Ignoring the three-particle correlations, we find directly from definition (2)

$$\Sigma^{\lessgtr}(\mathbf{p},\omega)=\int G^{\lessgtr}(\mathbf{p}',\omega')F^{\lessgtr}(\mathbf{p}'-\mathbf{p},\omega'-\omega)[V(\mathbf{p}'-\mathbf{p})]^2\frac{d\mathbf{p}'}{(2\pi)^3}\frac{d\omega'}{2\pi}.$$

Using

$$F^{\lessgtr}(\mathbf{p},\omega)=V^{-1}\sum_n|\langle 0|\rho(\mathbf{p})|n\rangle|^2 2\pi\delta(\omega\mp\omega_{n0})$$

and[9]

$$\int\frac{d\omega}{2\pi}[iG^{>}(\mathbf{p},\omega)-iG^{<}(\mathbf{p},\omega)]=1,$$

we find from $\langle .|\rho|n\rangle \sim \omega_c^{-1/2}$ in the limit $\omega_c \to 0$

$$\int \Gamma \frac{d\omega}{2\pi} = \int F(\mathbf{q}, 0)[V(\mathbf{q})]^2 \frac{d\mathbf{q}}{(2\pi)^3}. \tag{4}$$

At equilibrium, we find, using a relation from Ref. 10,

$$\int \Gamma d\omega \sim \int F(\mathbf{r})d\mathbf{r} \int [V(\mathbf{q})]^2 d\mathbf{q} \sim (T - T_c)^{-1}. \tag{5}$$

The kinetic evolution leading to fragmentation may be thought of as a monopole vibration of the nuclear density and may be described by a random phase equation.[11] The collision term would then determine the width $\Gamma_{\rm RPA}$, or classically, $\gamma_{\rm RPA} = \hbar^{-1} B^{-1} \Gamma_{\rm RPA}$ (B is a mass coefficient) would be the damping coefficient of a monopole mode. The total energy would be $E_{\rm tot} = E_{\rm in} + E_{\rm coll}$, and the decay time of the collective energy, $\Gamma_{\rm RPA}^{-1}$, would give us the relaxation time scale. Like the collision term in the ∞ system in (3), the width $\Gamma_{\rm RPA}$ increases with decreasing frequency of the monopole vibration, ω_M, and in the limit $\omega_M \to 0$ we have the curious result[12]

$$\lim_{\omega_M \to 0} \gamma_{RPA} = \gamma_W = m\rho\bar{v}R^4 \tag{6}$$

($\bar{v}$ is the average fermion velocity).

Here γ_W represents a single-body friction coefficient, which corresponds to the friction of the nuclear medium as a "wall" moves through it. So far, the single-body dissipation has been studied exclusively in deep inelastic reactions in the region <20 MeV/n above the barrier and in fission. The experimental value is an order of magnitude below γ_W. It is possible that γ_W will be reached first near the spinodal region. Correspondingly, $\Gamma_{\rm RPA} \sim \hbar(\bar{v}/R)$ might lead to a thermalization in a time $\tau \sim 0.5 \times 10^{-22}$ s, which would correspond to the hydrodynamic limit.[7] We note in conclusion that for a phase transition to occur we would need density fluctuations of the critical amplitude, and the decay mechanism discussed here would prevent a transition, causing an ordered motion to become random.

[1]B. Friedman and V. R. Pandharipande, Nucl. Phys. A **361**, 502 (1981).
[2]G. Röpke *et al.*, Phys. Lett. B **119**, 12 (1982).
[3]J. P. Bondorf *et al.*, Nucl. Phys. B **443**, 321 (1985).
[4]G. D. Westfall *et al.*, Contribution to Hirschegg-Workshop, 1990.
[5]H. S. Köhler and B. S. Nilsson, Nucl. Phys. A **417**, 541 (1984).
[6]E. Suraud *et al.*, Phys. Lett. B **229**, 359 (1989).
[7]H. S. Köhler, Nucl. Phys. A **378**, 159 (1982).
[8]L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1964)].
[9]P. Danielewicz, Ann. Phys. (N.Y.) **152**, 239 (1984).
[10]E. M. Lifshitz and L. P. Pitaevskiĭ, *Statistical Physics*, Pergamon, Oxford, 1980.
[11]A. Sustich and J. Wambach, Phys. Lett. B **218**, 417 (1989).
[12]C. Yannouleas, Nucl. Phys. A **489**, 91 (1988).

Translated by D. Parsons