Critical current of the Josephson junctions with Abrikosov vortices

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The average critical current of a Josephson junction is calculated as a function of the magnetic field at various concentrations of the Abrikosov vortices.

In Josephson junctions, whose sides are type-II superconductors, the Abrikosov vortices can be captured at right angles to the plane of the contact.[1-3] Such vortices are pinned at defects and their axes can, as indicated in Ref. 3, become distorted and can strongly suppress the critical current in the distortion region (because of the presence of a local magnetic field which accounts for the additional dependence of the Joseph-

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son phase on the coordinate). Such a suppression of the critical current for a single Abrikosov vortex in the absence of a magnetic field was observed in Ref. 3. In our experimental study we have found the critical current of a Josephson junction for different concentrations of the Abrikosov vortices in the presence of a parallel magnetic field.

Let us assume that a Josephson contact (the contact size is $L < \lambda_j$, where λ_j is the Josephson penetration depth) is situated in an external magnetic field H parallel to the plane of the contact, and that N distorted Abrikosov vortices are trapped in the contact. The critical current is given by

$$I_c^2 = j_0^2 \left| \int d^2\vec{\rho} \exp[i\varphi(\vec{\rho})] \right|^2, \tag{1}$$

where j_0 is the critical current density, and the integration is carried out over the entire surface area of the contact. The phase difference φ depends on the external field and on the coordinates $\vec{\rho}_i = (x_i, y_i)$ of the Abrikosov vortices in the Josephson contact

$$\varphi = \sum_{i=1}^{N} \varphi(\vec{\rho} - \vec{\rho}_i) + \frac{2\pi\phi x}{\phi_0 L} \qquad \sin\varphi = \frac{at\sin\theta}{\left[\left(t^2 - \frac{a^2}{4}\right)^2 + a^2t^2\sin^2\theta\right]^{1/2}}; \quad t = |\vec{\rho} - \vec{\rho}_i|, \tag{2}$$

where $\phi = 2HL\lambda$, L is the contact size, and a is the distance between the points at which the distorted vortex enters and leaves the Josephson contact. Figure 1 shows how the phase difference φ is determined. To determine the average current of the contact, I_c^2, we will average expression (1) over the different positions of the Abrikosov vortices. Assuming that these vortices are distributed randomly in the plane of the contact, we find the following expression by analogy with the procedure in Ref. 4:

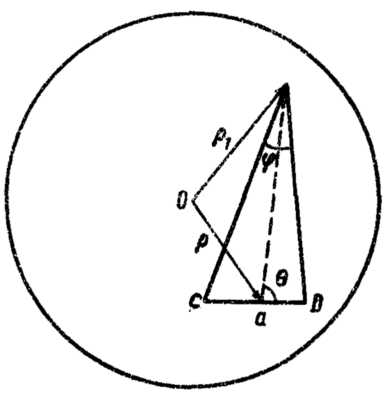

FIG. 1. Evaluation of the phase φ as a function of the coordinate for the Josephson contact with an Abrikosov vortex. C and D are the points at which the distorted vortex enters and leaves the Josephson contact. O—Center of the Josephson contact (L is the radius of the disk).

$$I_c^2 = j_0^2 \int d^2\vec{\rho}_1 \int d^2\vec{\rho}_2 \exp[\frac{i2\pi\phi}{\phi_0 L}(x_1 - x_2)] \{\int \frac{dxdy}{S} \int_0^{2\pi} \frac{d\theta}{2\pi}$$

$$\times \exp i[\varphi(\vec{\rho}_1 - \vec{\rho}) - \varphi(\vec{\rho}_2 - \vec{\rho})]\}^N =$$

$$= j_0^2 \int d^2\vec{\rho}_1 \int d^2\vec{\rho}_2 \exp\{\frac{i2\pi\phi}{\phi_0 L}(x_1 - x_2) + n \int d^2\vec{\rho} \int_0^{2\pi} \frac{d\theta}{2\pi}$$

$$\times \{e^{i[\varphi(\vec{\rho}_1 - \vec{\rho}) - \varphi(\vec{\rho}_2 - \vec{\rho})]} - 1\}\}, \quad (3)$$

where n is the concentration of the Abrikosov vortices in the Josephson contact. Evaluating the integral in the argument of the exponential function in Eq. (3), we find

$$\overline{I_c^2} = j_0^2 \int d^2\vec{\rho}_1 d^2\vec{\rho}_2 \exp\{\frac{i2\pi\phi}{\phi_0 L}(x_1 - x_2) - \pi n a^2 \ln(L/a)$$

$$+ \frac{na^2}{2} \int d^2\vec{\rho} \frac{\cos\alpha}{|\vec{\rho}_1 - \vec{\rho}||\vec{\rho}_2 - \vec{\rho}|}\}, \quad (4)$$

where α is the angle between the vectors $(\vec{\rho}_1 - \vec{\rho})$ and $(\vec{\rho}_2 - \vec{\rho})$. In the derivation of Eq. (4) we took into account that for small concentrations of the Abrikosov vortices the principal component of the current comes from $\vec{\rho}_1$ and $\vec{\rho}_2$ such that $|\vec{\rho}_1 - \vec{\rho}_2| \gg a$. Finally, we can write the critical current in the form

$$\overline{I_c^2} = j_0^2 \int d^2\vec{\rho}_1 \int d^2\vec{\rho}_2 f(|\vec{\rho}_1 - \vec{\rho}_2|) \exp[\frac{i2\pi\phi}{\phi_0 L}(x_1 - x_2)],$$

where

$$f(z) = \begin{cases} (a/z)^\gamma & z \gg a \\ 1 & z \ll a \end{cases} \qquad \gamma = \pi n a^2. \quad (5)$$

Assuming that the sample is a disk of radius L, we reduce Eq. (5) (using a Fourier transform) to the form

$$\overline{I_c^2} = j_0^2 (2\pi)^2 L^2 \int \frac{dq}{q} J_1^2(qL) J_0(qx) \int_0^\infty x f(x) J_0(\frac{2\pi\phi x}{\phi_0 L}) dx, \quad (6)$$

where $J_1(z)$ and $J_0(z)$ are the Bessel functions.

Expression (6) can be changed to the form

$$\overline{I_c^2} = 8\pi L^4 j_0^2 \int_0^1 dt (1 - t^2)^{1/2} \int_0^{2t} x f(xL) J_0(\frac{2\pi\phi x}{\phi_0}) dx. \quad (7)$$

In deriving Eq. (7) we made use of the integral

$$\int dq J_1^2(qL) J_1(qx) = \begin{cases} 0 & x > 2L \\ (1 - \frac{x^2}{4L^2})^{1/2} & x < 2L \end{cases} .$$

In the region of small concentrations $\gamma \ll 1$ the maximum values of $x \gg a$, and we rewrite expression (7) in the form

$$\overline{I_c^2} = j_0^2 8\pi L^4 (\frac{a}{L})^\gamma \int_0^1 dt (1-t^2)^{1/2} \int_0^{2t} x^{1-\gamma} J_0(\frac{2\pi\phi x}{\phi_0 L}) dx. \tag{8}$$

In the region of small magnetic fields ($\phi/\phi_0 \ll 1$) the Bessel function can be replaced by 1 and we find

$$\overline{I_c^2} = j_0^2 \frac{(8\pi)^{3/2}}{(2-\gamma)} L^4 (\frac{a}{2L})^\gamma \frac{\Gamma(\frac{3-\gamma}{2})}{\Gamma(3-\gamma)}. \tag{9}$$

In the opposite case of large magnetic fields ($\phi/\phi_0 \gg 1$), making use of the asymptotic behavior of the Bessel function at large values of the argument, we find

$$\overline{I_c^2} = j_0^2 8\pi L^4 (\frac{a}{2L})^\gamma \{ \frac{\gamma\pi}{4} \frac{\Gamma(1-\frac{\gamma}{2})}{\Gamma(1+\frac{\gamma}{2})\alpha^{2-\gamma}} + \frac{1}{\alpha^3} \left[\frac{\Gamma(\frac{3-\gamma}{2})}{\Gamma(\frac{1+\gamma}{2})\alpha^{-\gamma}} - \frac{\sin 2\alpha}{2} \right] \}; \ \alpha = \frac{2\pi\phi}{\phi_0}. \tag{10}$$

The first term in Eq. (10) describes the deviation from the Fraunhofer dependence due to the fluctuations of the Josephson phase in the contact. This small deviation at small values of γ decreases with increasing magnetic field (in contrast with the constant "pedestal" which appears because of the fluctuation of the critical current). We note that the critical current at nonzero magnetic fields behaves nonmonotonically with increasing concentration of the Abrikosov vortices (in contrast with the behavior of the current at $H=0$): when $\gamma = a/2L \ll 1$, the current reaches the maximum value. Equations (9) and (10) describe well the experimental dependences of the critical current on the magnetic field, which were presented in Ref. 2.

At large concentrations of the Abrikosov vortices ($\gamma \gg 1$), the principal component in Eq. (7) is $x \sim a$ and the critical current no longer depends on n: $\overline{I_c^2} \sim j_0^2 L^2 a^2$.

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