Hyperbolic strings, bosonized ghosts, and fermions

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A new type of string is introduced. In the Fok space of such strings one can fit Fok spaces of standard boson, fermion, and heterotic strings.

1. After the appearance of Refs. 1 it became clear that string theories can be formulated directly in the $d=4$ space $R^{3,1}$. Different string theories correspond to different points from modulus space M_4. We have $M_4^b = SO(22,22)/SO(22) \times SO(22)$ for boson strings, $M_4^f = SO(6,6)/SO(6)\times SO(6)$ for fermion strings, and $M_4^h = SO(22,6)/SO(22)\times SO(6)$ for heterotic strings.[2]

We obtain an infinite number of string theories in $R^{3,1}$, and it is not clear just which of them should be taken as the basic theory. The differences among the theories actually stem from differences in the lattices $\Lambda_{q,0}$ which generate the "internal" tori $T^{q,0} = R^{q,0}/\Lambda_{q,0}$. All such lattices are incorporated in a single Lorentzian lattice $\Lambda_{q+1,1}$. The transformation $T^{q,0}$ to the Lorentzian torus $T^{q+1,1} = R^{q+1,1}/\Lambda_{q+1,1}$, corresponds to the introduction of an additional time variable in the inner space of theories of the Kaluza–Klein type. This idea was proposed by Sakharov[3] and developed in Refs. 4. Using the additional time dimension, we introduce strings which we call "hyperbolic."

2. We consider a closed boson string in $R^{3,1}$ with $q+2$ compact bosons from the torus $T^{q+1,1}$. We call such a string, with the action

$$S = \frac{1}{2\pi} \int d^2\xi \sqrt{-G}\left[g_{AB} G^{\alpha\beta} \frac{\partial X^A}{\partial \xi^\alpha} \frac{\partial X^B}{\partial \xi^\beta} + \frac{1}{2} R g_{AB} X^A Q^B\right] \tag{1}$$

a "hyperbolic string." Here $\xi^1 = \tau$, $\xi^2 = \sigma$, $G_{\alpha\beta}$ is the metric on the world surface of the string; $G = \det(G_{\alpha\beta})$, $\alpha,\beta = 1,2, X^A$ are the coordinates of the target space $R^{3,1}\times T^{q+1,1}$, $A = 1,\ldots,q+6$, $g_{AB} = \mathrm{diag}(-+\ldots+-)$ is the metric on $R^{3,1}\times T^{q+1,1}$, and R is the scalar curvature for $G_{\alpha\beta}$, $Q^A = \mathrm{const}$.

The last term in (1) does not contribute to the equations of motion for X^A, but it does contribute to the corresponding operators L_n^A of Virasoro algebras.[5] Using a conformal gauge, and splitting $X^A(\xi^1,\xi^2) = X_+^A(\xi_+) + X_-^A(\xi_-)$ $(\xi_\pm = \xi^1 \pm \xi^2)$ up into left and right modes, we have

$$X_\pm^A(\xi_\pm) = q_\pm^A + p_\pm^A \xi_\pm + \frac{i}{2} \sum_{n\neq 0} \frac{1}{n} \alpha_{n\pm}^A \exp(-2in\xi_\pm). \tag{2}$$

For the operators $\hat{L}_n^\pm$ of Virasoro algebras $\mathscr{L}^\pm$ of the left and right sectors we have[5]

$$\hat{L}_n^{\pm} = \sum_{A=1}^{q+6} L_n^{A\pm},\ L_n^{A\pm} = \frac{1}{2} g_{AA}[\sum_m :\alpha^A_{-m\pm}\alpha^A_{(m+n)\pm}: - nQ^A\alpha^A_{n\pm} - \frac{1}{4}(Q^A)^2\delta_{n,0}]. \tag{3}$$

The commutation relations are standard.[2,5] For the central charges we have, respectively, $\hat{c}_{\pm} = \Sigma_{A=1}^{q+6} c_{A\pm}$, $c_{A\pm} = 1 - 3g_{AA}(Q^A)^2$.

The left and right sectors of the hyperbolic strings are identical, so we will discuss only the left sector; we will omit the $\pm$ from the expressions.

Following the usual quantization algorithm,[2,5] we obtain a Heisenberg algebra with the generators α_n^A, q^A, Id. We introduce the Fok space $\hat{H} = \mathbb{C}[R^{3,1} \oplus \Lambda_{q+1,1}] \otimes \mathrm{Sym}(\alpha^A_{-n})$ as the representation space of this algebra. Here $\mathrm{Sym}(\alpha^A_{-n})$ is the algebra of polynomials from α^A_{-n} $(n>0)$, and $\mathbb{C}[R^{3,1} \oplus \Lambda_{q+1,1}]$ is the algebra of functions of the type[6] $\exp(\gamma)$, $\gamma \in R^{3,1} \oplus \Lambda_{q+1,1}$.

We assume $\hat{c} = 0$ (we can arrange this condition by choosing q and Q^A appropriately), and we introduce the $\mathscr{L}$-invariant physical subspace $\hat{H}_0$ in Fok space $\hat{H}$:

$$\hat{H}_0 = \{\psi \in \hat{H} : \hat{L}_n \psi = 0,\ n \in \mathbb{Z}\}. \tag{4}$$

The space $\hat{H}_0$ corresponds to a transfer of the constraints $\hat{L}_n = 0$ to the quantum level.

Note that we have $Q^A = 0$ for standard bosons: $Q^A = \pm 1, \pm 2$ for "bosonized" superghosts β and γ; and $Q^A = \pm 3$ for bosonized ghosts b and c (Ref. 5).

3. We consider a closed Neveu–Schwartz–Ramond fermion string in $R^{3,1} \times T^{6,0}$. We know[2,5] that it is possible to bosonize fermions, ghosts, and superghosts of this theory by associating them with the tori $T^{5,0} = R^{5,0}/\Lambda_{5,0}$, $T^{1,0} = R^{1,0}/\Lambda_{1,0}$, and $T^{1,1} = R^{1,1}/\Lambda_{1,1}$, respectively. Here $\Lambda_{5,0}$ is a lattice of weights of the Lie algebra SO(10). A completely bosonized fermion string is therefore described by bosons $X^A = (X^\mu, X^I, Y^i)$ from the space $R^{3,1} \times T^{11,0} \times T^{1,1} \times T^{1,0}$; $\mu = 1,...,4$; $I = 1,...,11$; $i = 1,2,3$.

The action for a $d = 4$ bosonized fermion string has the form of (1) with $q = 12$, $Q^\mu = Q^I = 0$, $Q^1 = 1$, $Q^2 = 2$, $Q^3 = 3$. It is easy to see that we have $\hat{c} = 0$, and we can introduce a space $\hat{H}_0$ as in (4). An anomaly-free bosonized fermion string is thus the same as a hyperbolic string with $q = 12$, which we call a "fermion hyperbolic string."

Furthermore, the entire manifold of fermion strings in $d = 4$, which is described by the modulus space M_4^f, is incorporated in a single fermion hyperbolic string. This conclusion follows from the circumstance that the parameter $\lambda \in M_4^f$ is included among the characteristics of the lattice $\Lambda_{6,0}^\lambda$ and the circumstance that any Euclidean lattice $\Lambda_{6,0}^\lambda \oplus \Lambda_{5,0} \oplus \Lambda_{1,0}$, can be immersed in a unique canonical Lorentzian lattice $\Lambda_{13,1} \supset \Lambda_{6,0}^\lambda \oplus \Lambda_{5,0} \oplus \Lambda_{1,1} \oplus \Lambda_{1,0}$, which characterizes a fermion hyperbolic string. Correspondingly, the Fok space of any $d = 4$ fermion string can be found as a subspace in $\hat{H}_0$ by fixing the vacuum, $\lambda \in M_4^f$, and by fixing the class from $\Lambda_{5,0}$ (Refs. 2 and 5).

The choice of vector $\gamma \in R^{3,1} \oplus \Lambda_{6,0} \oplus \Lambda_{5,0} \oplus \Lambda_{1,1} \oplus \Lambda_{1,0}$ corresponds to a fixing of the vacuum $|\gamma\rangle = \exp\gamma$ of the string theory. In particular, the fixing $p \in \Lambda_{5,0} \oplus \Lambda_{1,1} \oplus \Lambda_{1,0}$ is called the fixing of the "picture" (a vacuum of fermions and ghosts). The "picture change" operator is[2] $X = \exp(p_{5,0})\exp(p_{2,1}) \in \mathbb{C}[\Lambda_{5,0} \oplus \Lambda_{2,1}]$

($\Lambda_{2,1} \equiv \Lambda_{1,1} \oplus \Lambda_{1,0}$). In other words, this operator simply consists of the elements of a group algebra for the discrete group $\Lambda_{5,0} \oplus \Lambda_{2,1}$ (Ref. 6).

4. We consider a hyperbolic string with $q = 24$ and the coordinates $X^A = (X^\mu, X^I, Y^\alpha, Z^i)$, $\mu = 1,...4$; $I = 1,...22$; $\alpha = 1,2$; $i = 1,2$. We set $Q^\mu = Q^I = Q^\alpha = 0$, $Q^1 = 3$, $Q^2 = 1$. It is easy to see that in this case the net central charge $\hat{c}$ of the Virasoro algebra is zero. We introduce $\hat{H}$ and $\hat{H}_0$.

The standard anomaly-free boson string in $d = 4$ corresponds to the target space $R^{3,1} \times T^{22,0}$. The lattice $\Lambda^\lambda_{22,0}$, which determines the torus $T_\lambda{}^{22,0}$, is characterized by $\lambda \in M^b_4$. We immerse $\Lambda^\lambda_{22,0}$ in a single canonical Lorentzian lattice $\Lambda_{25,1} \supset \Lambda^\lambda_{22,0} \oplus \Lambda_{1,1} \oplus \Lambda_{2,0}$, which specifies the torus $T^{25,1} = R^{25,1}/\Lambda_{25,1}$. We find that any standard theory of a $d = 4$ closed boson string is incorporated in a single anomaly-free boson hyperbolic string with $q = 24$ and with the values of Q^A given above.

5. We know that standard $d = 4$ heterotic strings are found by combining the left sector of a boson string in $R^{3,1} \times T^{22,0}$ and the right sector of a superstring in $R^{3,1} \times T^{6,0}$. It follows from the discussion that any heterotic string can be immersed in a single asymmetric hyperbolic string with the left sector from a boson hyperbolic string and the right sector from a fermion hyperbolic string. The "projection" onto a fixed standard $d = 4$ heterotic string can be understood as the fixing of the vacuum from the modulus space M^h_4 of heterotic strings.

6. A characteristic feature of hyperbolic strings is the presence of an additional time dimension, whose introduction in the framework of a field limit of strings was proposed by Sakharov.[3] The possibility of a more general string theory with a Lorentzian inner torus and a corresponding Lorentzian Kac–Moody algebra was also pointed out by Witten.[7] The results derived above suggest that further research on hyperbolic strings might be worthwhile.

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