Critical behavior of fracture stress in randomly inhomogeneous composites near percolation threshold

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In extremely inhomogeneous composites consisting of "hard" and "soft" phases, there may be a situation in which the deformation of the composite is determined by one of the phases, while fracture is determined by the other. The characteristics of the critical behavior of the concentration dependence of the fracture stress are found.

The effective kinetic coefficients (Refs. 1–3, for example)—the spectral density of the $1/f$ noise,[4] the elastic constants[4] and the fracture stress[6]—exhibit a critical behavior near the percolation threshold, as is well known. The basic characteristics of this behavior are the critical exponents. Sornette[6] has used models of the NLB type[7,8] to determine the critical exponents. In those models, the deformation and fracture of only the rigid phase are taken into account at $p>p_c$ (p is the concentration of the phase which is the good conductor if we are dealing with kinetic phenomena, or it is the concentration of the rigid phase if we are dealing with the elastic properties or mechanical fracture). Only the "elastic" phase is taken into account at $p<p_c$. It is shown below that when the two phases are taken into account at the same time, one may find a behavior and critical exponents which are different from those found in Ref. 6.

The deformation of the rigid and elastic phases can be taken into account simultaneously by using a hierarchical weak-link model,[9] based on the relationship between the geometry of the weak links and the concentration:

$$\frac{s_b}{l_b L} \sim \tau^t, \quad \frac{s_i}{l_i L} \sim |\tau|^{-q}, \tag{1}$$

where $\tau = (p-p_c)/p_c$.

The weak-link model has made it possible to describe many kinetic phenomena near the percolation threshold[10,11] and the concentration dependence of the elastic moduli and the Poisson ratios.[12,13]

Let us use the weak-link model to determine the critical behavior of the fracture stress of randomly inhomogeneous media as they are extended (Fig. 1). The basic deformation of the interlayer consists of a compression and a displacement $(l_i/\sqrt{s_i} \ll 1)$, while that of the bridge is a bending $(\sqrt{s_b}/l_b \ll 1)$. We first consider the case $p<p_c$. Denoting by $\mathscr{F}$ the force applied to a characteristic volume L^3 $(L \sim a_0|\tau|^{-\nu})$, we see that essentially the same force is applied to the interlayer

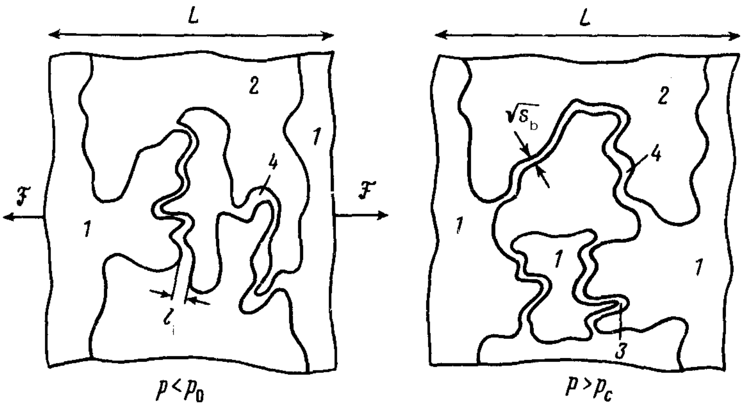

FIG. 1. Hierarchical weak-link model. L^3—Characteristic volume of the randomly inhomogeneous medium, where $L \approx \xi \approx a_0|\tau|^{-\nu}$; ξ—correlation length; ν—its critical exponent. 1) First (rigid) phase; 2) second (elastic) phase; 3) interlayer of a second phase, on which most of the load applied to the sample is concentrated in the case $p<p_c$; 4) bridge of first phase. l_i and s_i—Thickness and area, respectively, of the interlayer; l_b and s_b—length and cross-sectional area, respectively, of the bridge. Here $l_i \approx a_0$, $s_b \approx a_0^2$.

(phase 2) in a first approximation in the ratio of the Young's moduli of the elastic and rigid phases, $E_2/E_1 \ll 1$. If this force is of such a magnitude that fracture begins, then we have $\sigma_{c_2} = \mathscr{F}/s_i = (\mathscr{F}/L^2)(L^2/s_i) \approx \sigma_f|\tau|^{q-\nu}$, where σ_{c_2} is the fracture stress for phase 2, and σ_f is the fracture stress for the inhomogeneous medium as a whole. We thus have

$$\sigma_f \approx \sigma_{c_2} |\tau|^{F_1}, \quad F_1 = \nu - q. \qquad (2)$$

It is assumed here that the appearance of a crack (the destruction of a structural element) leads to the destruction of the entire material.

The deformation of the interlayer is accompanied by a simultaneous deformation (bending) of the bridge. Denoting by M_{c_1} the threshold moment of force at which the bridge is fractured, we have $\mathscr{F} \times L \approx M_{c_1}$, and thus

$$\sigma_f \approx \frac{M_c}{a_0^3} \tau^{F_2}, \quad F_2 = 3\nu. \qquad (3)$$

If $\sigma_{c_2} \tau^{\nu - q} > M_c \tau^{3\nu}/a_0^3$, the fracture of the material as a whole thus occurs as a result of the fracture of the interlayer, and the critical exponent is $F = F_1 = \nu - q$. If the opposite inequality holds, then we have $F = F_2 = 3\nu$.

Corresponding arguments in the case $p > p_c$ lead to $F = F_3 = 3\nu$ [if $M_c \tau^{3\nu}/a_0^3 > \sigma_{c_2} E_1 \tau^{t+3\nu}/(E_2 + 2\mu_2)$, where μ_2 is the shear modulus of phase 2; the bridge is fractured first.] This result is precisely the same as the value found for F in Ref. 5. Under the opposite inequality we find $F = F_4 = t + 3\nu$.

These arguments are valid for so-called discrete percolation, i.e., for media in which there exists a minimum length of a structural element, a_0. A generalization to the case of a continuum percolation (see, for example, the "swiss-cheese"[14] and blue-cheese[15] models) leads to an even greater density of particular cases and critical exponents and deserves a separate study.

[1] A. L. Efros and B. I. Shklovskiĭ, Phys. Status Solidi (b) **76**, 475 (1976).
[2] A. M. Dykhne, Zh. Eksp. Teor. Fiz. **59**, 110 (1970) [Sov. Phys. JETP **32**, 63 (1970)].
[3] B. I. Shklovskiĭ, Zh. Eksp. Teor. Fiz. **72**, 288 (1977) [Sov. Phys. JETP **45**, 152 (1977)].
[4] G. Rammal, J. Phys. (Paris) Lett. **46**, L129 (1985).
[5] P. G. De Gennes, J. Phys. (Paris) **37**, L1 (1976).
[6] D. Sornette, Phys. Rev. B **36**, 8847 (1987).
[7] A. S. Skal and B. I. Shklovskiĭ, Fiz. Tekh. Poluprovodn. **8**, 1586 (1975) [Sov. Phys. Semicond. **8**, 1029 (1975)].
[8] Y. Kantor and I. Webman, Phys. Rev. Lett. **52**, 1891 (1984).
[9] A. E. Morozovskiĭ and A. A. Snarskiĭ, Zh. Eksp. Teor. Fiz. **95**, 1844 (1989) [Sov. Phys. JETP **68**, 1066 (1989)].
[10] A. A. Snarskiĭ, Fiz. Tekh. Poluprovodn. **21**, 1877 (1987) [Sov. Phys. Semicond. **21**, 1136 (1987)]; Fiz. Tekh. Poluprovodn. **22**, 2073 (1988) [Sov. Phys. Semicond. **22**, 1314 (1988)].
[11] A. E. Morozovskiĭ and A. A. Snarskiĭ, Fiz. Tekh. Poluprovodn. **23**, 1220 (1989) [Sov. Phys. Semicond. **23**, 762 (1989)].
[12] A. A. Snarskiĭ, Ukr. Fiz. Zh. **33**, 1036 (1988).
[13] A. E. Morozovskiĭ and A. A. Snarskiĭ, Preprint, Institute of Metal Physics, September 11, 1990.
[14] B. I. Halperin and S. Feng, Phys. Rev. Lett. **54**, 2391 (1985).
[15] D. Sornette, J. Phys. (Paris) **49**, 1365 (1988).

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