Generalization of gauge RVB theory to a non-1/2 filling

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(Submitted 30 May 1990; resubmitted 14 July 1990)
Pis'ma Zh. Eksp. Teor. Fiz. **52**, No. 4, 880–882 (25 August 1990)

A generalization of a gauge theory of antiferromagnetism to the case of a t–J model with a filling $\nu \sim 1/2$ is proposed.

Several versions of an RVB gauge theory[1-3] have recently been proposed, with both Abelian and non-Abelian gauge groups. It has been shown in the mean field approximation that the effective long-wavelength Lagrangian with a filling $\nu = 1/2$ contains a Chern–Simons (CS) term and that excitations have fractional statistics.

A direct generalization of the theory to the case $\nu \neq 1/2$ is not possible, since doping disrupts the gauge invariance. With $\nu \sim 1/2$, however, one can hope that a gauge-invariant model, in which doping effects are dealt with phenomenologically [through the incorporation of an NNN interaction and through the choice of the SU(N) representation which simulates the filling ν], will yield qualitatively correct results. Below we examine a particular example of this model. We show that as the parameter of the model, λ, is varied (this parameter characterizes the NNN interaction in the mean-field approximation), there is a cascade of statistics-transmuting phase transitions of a "devil's-staircase" type.

Let us formulate the model. We consider an antiferromagnet on Z^2 with spin operators X from the $l\,\overline{\omega}_f$ representation of the algebra $u(N)$, which is specified by the senior weight $(l,\dots,l,_{f\text{ times}}0,\dots0)$, with $f = \nu N$ and $N \gg l$.

$$H = J\,\mathrm{tr}\sum_{x\in Z^2}\sum_{j=1,2} X(x)X(x+\hat{e}_j) + J'\,\mathrm{tr}\sum_{x\in Z^2}\sum_{\epsilon_{1,2}=\pm 1} X(x)X(x+\epsilon_1\hat{e}_1+\epsilon_2\hat{e}_2). \quad (1)$$

Hamiltonian (1) incorporates NNN terms, which reflect the effect of the doping and which, as we will see, cause a substantial restructuring of the mean-field picture. The representation $\overline{l\omega_f}$ has been chosen because we wish to obtain a filling ν in the limit $N\to\infty$. This construction was proposed (in a slightly different context) by Wiegmann and Khveshchenko[3]). Following them,[3] we introduce a bilinear representation for the operators X in terms of fermion creation and annihilation operators:

$$X_{nm}(x)=C_n^{\alpha+}(x)C_m^{\alpha}(x);\ n,m=1,\dots,N:\ \alpha=1,\dots,l;\ C_1(\overline{l\omega_f})=lf.$$

In other words, there are f fermions at a node; the corresponding filling is $\nu=f/N$. The latent degrees of freedom of Hamiltonian (1) are therefore activated in the representation $\overline{l\omega_f}$, and they become dynamic in the mean-field approximation.

In the mean field approximation ($N\to\infty$, $\nu=f/N=\text{const}$, $l=\text{const}$) we find the Hamiltonian

$$\check{H}=\sum_{x\in\mathbb{Z}^2}\sum_{j=1,2}C_n^{\alpha+}(x)\chi^{\alpha\beta}_{x,x+\hat{e}_j}C_n^{\beta}(x+\hat{e}_j)$$
$$+\sum_{x\in Z^2}\sum_{\epsilon_{1,2}=\pm1}C_n^{\alpha+}(x)\tilde{\chi}^{\alpha\beta}_{x,x+\epsilon_1\hat{e}_1+\epsilon_2\hat{e}_2}$$
$$\times C_n^{\beta}(x+\epsilon_1\hat{e}_1+\epsilon_2\hat{e}_2), \qquad (2)$$

which describes hops along the edges and diagonals of a square lattice in a gauge field $[\chi,\tilde{\chi}\in\mathrm{GL}(l)]$. In the case of a uniform flux phase, the mean field values of χ and $\tilde{\chi}$ are

$$\tilde{\chi}_{xy}=\chi\exp(iA_{xy}),\ \prod_{\langle xy\rangle\in\partial P}\exp(iA_{xy})=e^{2\pi i\phi},$$
$$\tilde{\chi}_{xy}=\frac{\lambda\chi}{2}\exp(i\tilde{A}_{xy}),\ \prod_{\langle xy\rangle\in\partial\tilde{P}}\exp(i\tilde{A}_{xy})=e^{4\pi i\phi}, \qquad (3)$$

where $\phi=p/q$ is the flux through a plaquette.

As the parameter λ is varied from 0 to 1, the spectrum of modified Hofstädter problem (2), (3) is restructured in such a way that for a given $\phi=p/q$ and for fillings $\nu=r/q$ there exist at certain $r\in\{1,\dots,q\}$ some critical values $\lambda_c(\phi,\nu)$ such that at $E=\varepsilon_F(\phi,\nu)$ two bands touch. The Fermi surface degenerates to q isolated points, near which linearized Hamiltonian (2) describes Dirac fermions with a mass $m\sim\Delta\lambda\equiv\lambda-\lambda_c$. After the fluctuations of the SU(l) gauge field are taken into account, and an integration is carried out over fermions, we find an anomalous component of the effective Lagrangian of the form $q\,\mathrm{sign}(\Delta\lambda)\times(\mathrm{CS})$, in complete agreement with the general theory.[4] According to the latter, when bands touch, one of them gives the other a part of a Chen class, which is equal to the basis class from $H^2(T^2_*,\mathbb{Z})$; in other words, $\sigma_{xy}\to\sigma_{xy}\pm q$.

It is interesting to look at the result of the restructuring of the spectrum upon a variation in λ from the global standpoint. We construct a series of graphs $\overline{E}_\nu(\phi)$, where $\overline{E}$ is the total energy. On an individual graph $\overline{E}_\nu(\phi)$ there are cusps which

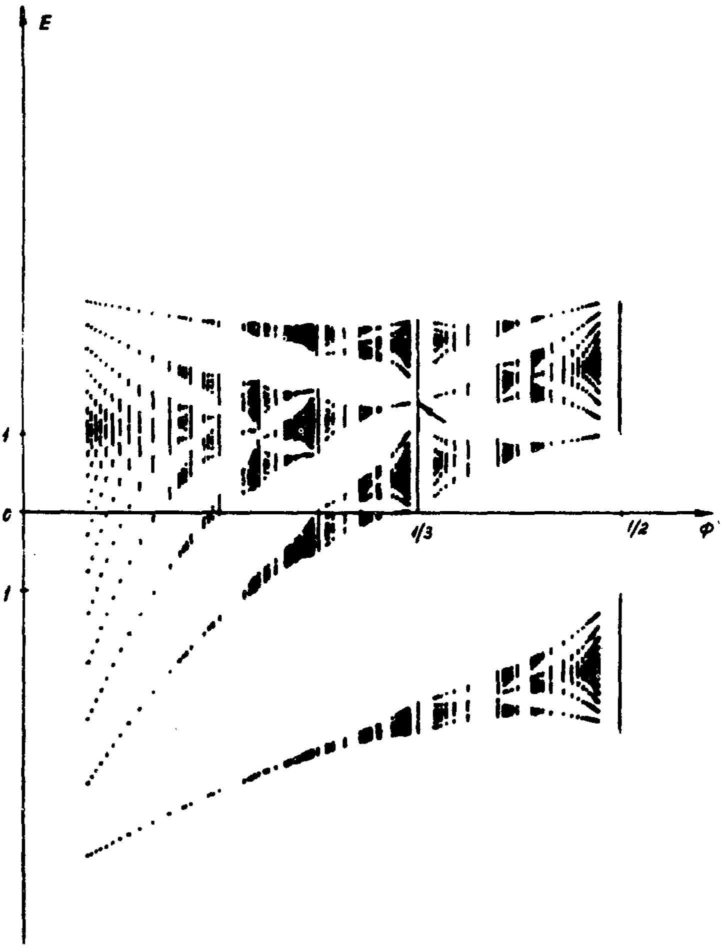

FIG. 1. Modified spectrum of the Hofstädter problem for $\lambda = \lambda_c(1/3,1/3)$. The arrow shows the point at which the bands touch.

group into trajectories when the entire family $\{\overline{E}_\nu(\phi)\}$ as a whole is considered (Ref. 5, for example). With $\lambda = 0$, there are two most prominent sequences of such trajectories, which correspond to $\nu = n\phi$(I) and $1 - \nu = n\phi$(II), for which we have $\sigma_{xy} = n$. With $\lambda = 1$, only a single sequence remains. The change in the situation results from a devil's staircase of statistics-transmuting phase transitions, which have the following local appearance: We assume for definiteness $0 < \phi < 1/2$ and $\nu > 1/2$. We reduce λ from 1 to 0. In the process, the trajectories of sequence (I) become progressively less prominent, while trajectories (II) become progressively more prominent. At points (ϕ,ν) at which the continuations of the (I) and (II) trajectories intersect with $\lambda = \lambda_c(\phi,\nu)$ a phase transition occurs. In the course of this transition, σ_{xy} changes by q. In general, there are thus also changes in the level $k = 2|\sigma_{xy}|$ of the effective *WZNW* theory and in the statistics of the excitations. This statistics is determined by the conformal weight of the corresponding Primar field (the case $\nu = 1/2$ is an exception; in this case, the touching of bands is not accompanied by a transmutation of the statistics). The phase transitions are ordered in n; as $\lambda \rightarrow 0$, they come closer together. In addition, there are transitions associated with nonprincipal sequences of cusp trajectories.

Figure 1 illustrates the situation with a modified spectrum of the Hofstädter problem for $\lambda = \lambda_c(1/3,1/3) = 4 - 2\sqrt{3}$.

In summary, in this model the hierarchy of chiral spin liquids becomes restructured in a complicated way as the parameter λ is varied. The "fundamental" chiral spin liquid, corresponding to semions, is not affected by these deformations of the spectrum, however. The observed phase transitions are apparently a manifestation of a latent topological order of the model.

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Translated by D. Parsons