Current noise due to inelastic electron scattering above T_c in superconductors

K. É. Nagaev
Institute of Radioengineering and Electronics, Academy of Sciences of the USSR

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The electrical noise in excess of the equilibrium electrical noise has been calculated for temperatures above T_c in a current-carrying superconductor. This excess noise is associated with fluctuations of the electron energy distribution. The magnitude of this noise is $T/(T - T_c)$ times the noise associated with fluctuations of the electron temperature, because the fluctuations of the paraconductivity are determined by a narrow interval of electron energies.

Because of the long energy relaxation time of quasiparticles, effects associated with a deviation of these quasiparticles from equilibrium play an important role in metallic superconductors.[1] In particular, in the resistive states of these superconduc-

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tors fluctuations of the quasiparticle energy distribution give rise to a strong excess noise, i.e., a noise in excess of the equilibrium noise.[2] It is interesting to see what current noise would be produced by these fluctuations above T_c, in the paraconductivity region.

The temperature-dependent corrections to the conductivity of a normal metal near the superconducting transition are given by the standard diagrams discussed in Refs. 3–5. In those papers, the corresponding contributions were calculated for the case of an equilibrium distribution. For an arbitrary nonequilibrium electron distribution, the analytic expressions for the corrections to the current density in a field **A** with a frequency ω_0 are

$$\mathbf{j}_{AL} = -i\frac{\mathbf{A}}{c}\frac{\pi^3 N^3 D^2}{32T^2}\int\frac{d^3q}{(2\pi)^3}q^2\int\frac{d\omega}{2\pi}\times K^R(\omega,\mathbf{q})\left[K^R(\omega+\omega_0,\mathbf{q}) - K^R(\omega,\mathbf{q}) + K^A(\omega-\omega_0,\mathbf{q}) - K^A(\omega,\mathbf{q})\right]K^A(\omega,\mathbf{q}), \quad (1)$$

$$\mathbf{j}_{MT} = i\frac{\mathbf{A}}{c}\frac{\pi e^2 N}{4m^2}\int\frac{d^3q}{(2\pi)^3}\int\frac{d\omega}{2\pi}K^R(\omega,\mathbf{q})K^A(\omega,\mathbf{q})\int d^3r_1\int d^3r_2\int\frac{d\epsilon}{2\pi} \times\Big\langle G^A(\epsilon,\mathbf{r}',\mathbf{r}_1)\frac{\partial}{\partial\mathbf{r}_1}G^R(\epsilon+\omega_0,\mathbf{r}_1,\mathbf{r})\Big[G^R(\omega-\epsilon,\mathbf{r}',\mathbf{r}_2)\frac{\partial}{\partial\mathbf{r}_2}G'(\omega-\epsilon-\omega_0,\mathbf{r}_2,\mathbf{r}) + G(\omega-\epsilon,\mathbf{r}',\mathbf{r}_2)\frac{\partial}{\partial\mathbf{r}_2}G^A(\omega-\epsilon-\omega_0,\mathbf{r}_2,\mathbf{r})\Big]\Big\rangle_{\mathbf{q}}. \quad (2)$$

Here D is the electron diffusion coefficient, N is the Fermi density of states, the angle brackets mean an average over the positions of the impurities, and the index $\mathbf{q}$ on the angle brackets means that the Fourier transforms are taken in terms of the coordinate difference $\mathbf{r}-\mathbf{r}'$. The Keldysh Green's function[6] G' is expressed in terms of the retarded and advanced Green's functions and the nonequilibrium electron energy distribution $F(\epsilon)$ (Ref. 7):

$$G'(\epsilon,\mathbf{r},\mathbf{r}') = [G^A(\epsilon,\mathbf{r},\mathbf{r}') - G^R(\epsilon,\mathbf{r},\mathbf{r}')]F(\epsilon). \quad (3)$$

At equilibrium we have $F(\epsilon)=\tanh(\epsilon/2T)$. The retarded and advanced Cooper propagators are given by[3]

$$K^{R(A)}(\omega,\mathbf{q}) = [i/\lambda + \Pi^{R(A)}(\omega,\mathbf{q})]^{-1}, \quad (4)$$

where λ is the electron-phonon coupling constant, and

$$\Pi^{R(A)}(\omega,\mathbf{q}) = \int\frac{d\epsilon}{2\pi}\langle G^{R(A)}(\epsilon,\mathbf{r},\mathbf{r}')G'(\omega-\epsilon,\mathbf{r},\mathbf{r}')\rangle_{\mathbf{q}}. \quad (5)$$

Restricting the analysis to a dirty superconductor $(1/\tau \gg T)$, and linearizing expressions (1)–(5) in terms of fluctuations of the electron energy distribution, $\delta F(\epsilon) = F(\epsilon) - \tanh(\epsilon/2T)$, we find the following expression for the conductivity fluctuations:

$$\delta\sigma = \delta\sigma_{AL} + \delta\sigma_{MT} = 32e^2T^2D \int \frac{d^3q}{(2\pi)^3} \int \frac{d\epsilon}{2\pi} \frac{\epsilon F(\epsilon)}{[\epsilon^2 + (\Gamma/2 + Dq^2)](\Gamma + Dq^2)}$$

$$\times \left\{ \left[\frac{3Dq^2}{(\Gamma + Dq^2)^2} + \frac{1}{Dq^2}\right] \left[\frac{1}{\Gamma + Dq^2} + \frac{\Gamma/2 + Dq^2}{\epsilon^2 + (\Gamma/2 + Dq^2)^2}\right] - \frac{\Gamma/2 + Dq^2}{\Gamma + Dq^2} \frac{\epsilon^2 - 3(\Gamma/2 + Dq^2)}{[\epsilon^2 + (\Gamma/2 + Dq^2)]^2} \right\} . \quad (6)$$

Here $\Gamma = 8(T - T_c)/\pi$.

Let us examine the temperature region in which the energy relaxation time of the electrons, τ_ϵ, is long in comparison with the time scales of the paraconductivity: $\tau_\epsilon(T - T_c) \gg 1$. In this case, fluctuations of the paraconductivity adiabatically follow the fluctuations of the electron distribution. We also ignore the effect of superconducting fluctuations on the electron-phonon coupling. We can then use the semiclassical expression for the correlation function for fluctuations of the electron distribution. As we will see below, the noise is dominated by electrons with $\epsilon \ll T$. Accordingly, the correlation function has only a term which is local in the energy.[2] Using the Langevin equation for δF and the correlation function for the external fluxes for it, which are given in Ref. 2, we easily find that the Fourier transform of the correlation function of the fluctuations in the distribution function is

$$<< \delta F(\epsilon_1)\delta F(\epsilon_2) >>_\Omega = 4\tau_\epsilon(1 + \Omega^2\tau_\epsilon^2)^{-1}\delta(\epsilon_1 - \epsilon_2)/NV, \quad (7)$$

where V is the volume of the superconductor.

We consider a thin superconducting film of thickness d. We substitute correlation function (7) into (6). The logarithmic divergences which arise in the course of the integration over small values of q are cut off by incorporating the energy relaxation in the electron Green's functions. In the leading approximation in $\ln[\tau_\epsilon(T - T_c)]$, we find

$$S_\sigma(\Omega) = 2 << \delta\sigma(t_1)\delta\sigma(t_2) >>_\Omega$$

$$= \frac{3\pi^2}{2^9} \frac{T^5}{(T - T_c)^5} \ln^2[\tau_\epsilon(T - T_c)]\tau_\epsilon \frac{e^4/d^2}{NVT(1 + \Omega^2\tau_\epsilon^2)} . \quad (8)$$

Since the average thermodynamic square of the fluctuations of the electron temperature satisfies $\langle\langle \delta T_{el}^2 \rangle\rangle \sim T/NV$, the conductivity noise which we found is larger by a factor of $T/(T - T_c)$ than the noise associated with fluctuations of the electron temperature. The reason for this result is that the fluctuations of the electrical conductivity are determined by a narrow layer of electron energies with the width on the order of $T - T_c$.

The ratio of the spectral density of voltage fluctuations, S_U, corresponding to (8), on the one hand, to the Nyquist noise of the normal electrons, S_U^N, on the other, is $S_U/S_U^N = E^2VS_\sigma/4T\sigma_N$, where σ_N is the conductivity of the normal metal. The value of the voltage noise associated with conductivity fluctuations is limited by the maximum attainable electric field E, which is determined either from the condition for heating of electrons, $\delta T_{el} \sim \tau_\epsilon De^2E^2/T \sim T - T_c$, or from the condition for the suppression of the fluctuational superconductivity by the field: $De^2E^2/(T-T_c)^2 \sim T - T_c$. For a film thickness $d = 100$ Å, a mean free path on the order of the lattice constant, $T_c \sim 10$ K, and $T_c\tau_\epsilon \sim 10^2$, the maximum current noise at frequencies $\Omega \ll \tau_\epsilon^{-1}$ reaches a level on the order of the Nyquist noise even at $T - T_c \sim 1$ K. For metallic superconductors, this current noise could thus be observed over a temperature range far broader than that in which the fluctuational correction to the electrical conductivity can be observed. It would be interesting to see whether the temperature dependence in (8) is observed in the high-T_c superconductors. A resolution of this point would shed light on whether the superconductivity here is due to the BCS mechanism.

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