Polarized states of Andreev quasiparticle in *SFS* and *SF* junctions, where *F* is a ferromagnetic metal

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The spectrum of an Andreev quasiparticle in *SFS* and *SF* junctions is derived. Completely polarized discrete levels are predicted. The relationship with experiment is discussed.

The exchange field of a ferromagnetic film in contact with a superconductor causes a precession of the spins of the Cooper electrons, which in turn disrupts the singlet correlation. As a result, T_c is lowered substantially (for thin superconducting films), and the establishment of a single coherent state in an *SFS* junction (*S* is a superconductor, and *F* a ferromagnetic film) becomes impossible if $l_{s0} \gg v_0/h$ (l_{s0}) is the spin-orbit scattering length in the *F* region, v_0 is the Fermi velocity, and h is the exchange-interaction parameter) and if the thickness (d) of the ferromagnetic film exceeds a critical value $d_c = \beta v_0/h$, $(h/E_F)^{1/2} \ll \beta \leqslant \pi/4$, where E_F is the Fermi energy. The latter effect has apparently been observed in experiments[1] on a Pb–Fe–Pb

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system ($d_c \sim 5$ Å). A theoretical explanation of this effect was offered in Ref. 2. In the present letter we analyze the effect of the exchange field on the states of an Andreev quasiparticle[3] in pure *SFS* ($d < \beta v_0/h$) and *SF* [$d < \beta v_0/(2h)$] junctions at temperatures $T \ll T_c$.

We will demonstrate that discrete levels can be completely polarized, and we will discuss the relationship with experiment.

A ferromagnetic film fills the region $|z| \leqslant d/2$, while a semiconductor fills the regions $|z| > d/2$. The junction is uniform along the x and y directions. The modulus of the pairing potential Δ is constant over the entire S region (the validity of this model is discussed below). Quasiparticles with energies $0 < \epsilon < \Delta$ are described by the semiclassical Bogolyubov–de Gennes equations

$$\{\epsilon\tau_0\sigma_0 + iv_0\cos\theta\tau_3\sigma_0\frac{d}{dz} + h\Theta(d/2 - |z|)\tau_3\sigma_3$$
$$- \Delta[\Theta(z - d/2)e^{i\varphi/2} + \Theta(-z - d/2)e^{-i\varphi/2}]$$
$$\times\frac{1}{2}(\tau_1 + i\tau_2)i\sigma_2 + \Delta[\Theta(z - d/2)e^{-i\varphi/2}$$
$$+ \Theta(-z - d/2)e^{i\varphi/2}]\frac{1}{2}(\tau_1 - i\tau_1)i\sigma_2\}\Psi = 0. \quad (1)$$

Here $\tau_i\sigma_k$ is the direct product of Pauli matrices in Gor'kov–Nambu space (τ_i) and in spin space (σ_k), τ_0 and σ_0 are the corresponding unit matrices, $\Theta(z)$ is the unit step function (the "Heaviside function"), θ is the angle between the direction of $\vec{v}$ and the z axis, φ is the coherent phase difference, and $\Psi = (u_\uparrow, u_\downarrow, -v_\uparrow, v_\uparrow)$, where the functions $u_\alpha(z)$ and $v_\beta(z)$ are continuous at $z = \pm d/2$, decay as $z \to \pm\infty$, and satisfy normalization conditions. Note that the use of Eqs. (1), of lowered order, means that we are considering excitations for which the relation $|\cos\theta| \gg \max\{(T_c/E_F)^{1/2}, (h/E_F)^{1/2}\}$ holds. If this condition does not hold, the semiclassical approximation must not be used.

Equation (1) breaks up into two independent equations, for the spinors $\psi_+ = (u_\uparrow, v_\downarrow)$ and $\psi_- = (u_\downarrow, -v_\uparrow)$. From the condition under which there is a solution we find

$$\mathrm{tg}\left[\frac{\epsilon_\pm d}{v_0|\cos\theta|} + \mathrm{sgn}(\cos\theta)\frac{\varphi}{2} \pm \frac{hd}{v_0|\cos\theta|}\right] = \frac{\sqrt{\Delta^2 - \epsilon_\pm^2}}{\epsilon_\pm}, \quad (2)$$

where the $\pm$ correspond to the indices of the spinors ψ_+, ψ_-. Equation (2) for $h = 0$ is well known:[3,4] It gives the spectrum of an Andreev quasiparticle in an *SNS* junction. Values $\varphi \neq 0$ correspond to a current state. Here are the basic properties of (2) with $h = 0$ and $\varphi = 0$: There always exists at least one solution $0 < \epsilon_0 < \Delta$, and the total number of physical solutions is equal to the greatest integer in the ratio $\Delta d/\pi v_0|\cos\theta|$ plus 1. At $h > 0$ we have the spectrum of localized excitations in an *SFS* junction ($d < \beta v_0/h$) and in an *SF* junction ($\varphi = 0$, $d \to 2d < \beta v_0/h$), which we have been seeking.

We begin our analysis of (2) with the case of a narrow barrier, $d \ll \xi = v_0/\Delta$, and excitations which are propagating along the normal to the boundary ($\cos\theta = 1$). If $\varphi = 0$ and $h = 0$, there exists a unique, spin-degenerate level $\epsilon_0 = \Delta(1 - d^2/2\xi^2)$. Incorporating a weak exchange field leads to a splitting:

$$\epsilon_{0\pm} = \epsilon_o \mp hd^2/\xi^2, \quad h \ll \Delta. \tag{3}$$

With increasing h, the level ϵ_{0-} rises, while ϵ_{0+} descends. At $h = \Delta$, the level ϵ_0 is pushed out into the continuum ($\epsilon_{0-} = \Delta$). At $h = \pi v_0/(2d)$ there is the formal solution $\epsilon_{0+} = 0$, which signifies a disruption of the discrete spectrum. According to Ref. 2, however, at even weaker fields $h_c = \beta v_0/d$ the phase correlation of the superconductors is disrupted (a complete uncertainty regarding φ), and Eq. (2) cannot be used. In directions $|\cos| \leqslant 2hd/\pi v_0$, the discrete spectrum is disrupted even at $h < h_c$, and Eq. (2) does not describe any physical states. We can formulate the general result in this way: If the condition

$$d\xi^{-1} \leq hd/v_0 < \beta \tag{4}$$

holds, the only quasiparticles which localize in the region of the ferromagnetic barrier are those which have a positive spin orientation (ψ_+) and for which the condition $|\cos| > 2hd/(\pi v_0)$ holds. The energy of such quasiparticles is given by (2) with the plus sign.

If we now impose, along with (4), the condition $d \ll v_0/(hE_F)^{1/2}$, we find from (2)

$$\epsilon_{0+} = \Delta(1 - h^2 d^2/2v_0^2|\cos\theta|^2). \tag{5}$$

States like those in (5) were observed in Ref. 5 for the model of Ref. 6, in which the ferromagnet is described by the potential $V(z) = J\sigma_3\delta(z)$. Expression (5) yields the identification[1)] $J \equiv hd/2$, and it has a clear physical meaning, in contrast with the case with a model with a δ-function potential.

DeWeert and Arnold[5] have combined the concepts of the states in (5) with several additional assumptions (spin nonconservation during quasiparticle tunneling and the presence of ferromagnetic domains near the barrier) to explain the triple splitting of the tunneling characteristic of a symmetric Pb–$Ho(OH)_3$–Pb junction [$Ho(OH)_3$ is a ferromagnetic insulator with a Curie temperature ~ 2.5 K] observed in Ref. 7. It is our opinion that weakly split levels of the type in (3) may have been observed in Ref. 7. The suppression of Cooper pairing in the course of interactions with the barrier necessarily results in the formation of a potential well for them, with an effective width $\sim 2\xi$ and a depth $\sim 2T_S\Delta \ll \Delta$, where T_S is the exchange part of the tunneling probability.[6] In this interpretation, there is no need for further assumptions.[5]

Strictly speaking, the suppression of Cooper pairing must also be taken into consideration in *SFS* and *SF* junctions. A change in the profile of the potential well implies a shift of the levels. In general, on the other hand, the polarization effects discussed above should persist. The value of Δ can be held constant in superconducting electrodes with a thickness less than ξ. In this case, however, one must allow for the possibility of a pronounced lowering of T_c, to the point that the superconducting transition is completely suppressed.[1]

We conclude with an expression for the low-lying levels in the limit $h \ll \Delta$ and $\xi \ll d < d_c (SFS)$:

$$\epsilon_{n\pm} = \frac{\pi v_0 |\cos\theta|}{d} \left[n + \frac{1}{2} - \mathrm{sgn}(\cos\theta) \frac{\varphi}{2\pi} \right] \mp h,$$

$$|\cos\theta| > 2hd/\pi v_0, \quad n = 0, 1, 2 \ldots$$

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[1)] The process of taking the limit of a δ-function potential is analyzed rigorously in the Appendix to Ref. 2.

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